MAT 212, Quiz #14, Spring 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Use improved Euler's method to estimate the first three steps of the calculation for the ODE $y' = 3 + t - y, y(0) = 1$ using the step size $h = 0.05$.

$y_0 = 1$ $t_0 = 0$ $m = 3+0-1 = 2$ $t = 0 + .025 = .025$ $y = \frac{1}{2}(2)(.05) + 1 = 1.05$

$f = 3 + .025 - 1.05 = 1.975$ $y_1 = 1 + .05(1.975) = 1.09875$

$y_1 = 1.09875$ $t_1 = 0.05$ $m = 3 + .05 - 1.09875 = 1.95125$ $t = .075$ $y = \frac{1}{2}(1.95125)(.05) + 1.09875 = 1.14753125$

$f = 3 + .075 - 1.14753125 = 1.92746875$

$y_2 = 1.09875 + .05(1.92746875) = 1.195123438$

$y_2 = 1.195123438$ $t = .10$ $m = 3 + .1 - 1.195123438 = 1.904876563$

$t = .125$ $y = \frac{1}{2}(.05)(1.904876563) + 1.195123438 = 1.242745352$

$f = 3 + .125 - 1.242745352 = 1.882254648$

$\boxed{y_3 = 1.2892362}$

$y_3 = 1.195123438 + .05(1.882254648) = 1.2892362$

2. Repeat the calculation above using Runge-Kutta. but for one step

$k_{n1} = 3 + 0 - 1 = 2$ $k_{n2} = 3 + .025 - 1.05 = 1.975$ $k_{n3} = 3 + .025 - 1.049375 = 1.975625$

$t = .025$ $y = 1 + .025(2) = 1.05$ $t = .025$ $y = 1 + .025(1.975) = 1.049375$

$k_{n4} = 3 + .05 - 1.09878125 = 1.95121875$

$t = .05$ $y = 1 + .05(1.975625) = 1.09878125$

$$y_1 = 1 + .05\left(\frac{2 + 2(1.975) + 2(1.975625) + 1.95121875}{6}\right) = \boxed{1.098790573}$$

3. Use Runge-Kutta on $x' = 2x + ty, y' = xy, x(0) = 1, y(0) = 1$ for ~~three steps~~ one step (in each variable) with step size $h = 0.2$.

$\vec{x}(0) = \begin{pmatrix}1\\1\end{pmatrix}$ $t_0 = 0$ $x_0 = 1$, $y_0 = 1$

$k_{n1} = \begin{pmatrix}2(1) + (0)1\\(1)(1)\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix}$ $k_{n2} = \begin{pmatrix}2(1.2) + .1(1.1)\\(1.2)(1.1)\end{pmatrix} = \begin{pmatrix}2.51\\1.32\end{pmatrix}$ $k_{n3} = \begin{pmatrix}2(1.251) + .1(1.132)\\(1.251)(1.132)\end{pmatrix} = \begin{pmatrix}2.6152\\1.416132\end{pmatrix}$

$t = .1$ $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix} + .1\begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}1.2\\1.1\end{pmatrix}$ $t = .1$ $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix} + .1\begin{pmatrix}2.51\\1.32\end{pmatrix} = \begin{pmatrix}1.251\\1.132\end{pmatrix}$

$k_{n4} = \begin{pmatrix}2(1.52304) + .2(1.2832264)\\(1.52304)(1.2832264)\end{pmatrix} = \begin{pmatrix}3.30272528\\1.954405136\end{pmatrix}$

$t = .2$ $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix} + .2\begin{pmatrix}2.6152\\1.416132\end{pmatrix} = \begin{pmatrix}1.52304\\1.2832264\end{pmatrix}$

$$\left(\frac{.2}{6}\right)\left[\begin{pmatrix}2\\1\end{pmatrix} + 2\begin{pmatrix}2.51\\1.32\end{pmatrix} + 2\begin{pmatrix}2.6152\\1.416132\end{pmatrix} + \begin{pmatrix}3.30272528\\1.954405136\end{pmatrix}\right] = \begin{pmatrix}.5184375093\\.2808889712\end{pmatrix}$$

$$\begin{pmatrix}x_1\\y_1\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix} + \begin{pmatrix}.5184375093\\.2808889712\end{pmatrix} = \boxed{\begin{pmatrix}1.518437509\\1.280888971\end{pmatrix}}$$

Modified Euler': $y_{n+1} = y_n + hf\left[t_n + \frac{1}{2}h, y_n + \frac{1}{2}h\boxed{f(t_n, y_n)}^{M}\right]$

Runge-Kutta: $y_{n+1} = y_n + h\left(\frac{k_{n1}+2k_{n2}+2k_{n3}+k_{n4}}{6}\right),$

$$k_{n1} = f(t_n, y_n), k_{n2} = f\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_{n1}\right),$$

$$k_{n3} = f\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_{n2}\right), k_{n4} = f(t_n + h, y_n + hk_{n3})$$