

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Differentiate each function.

a. $y = e^x \cos x$

$$y' = e^x \cos x - e^x \sin x$$

$$= e^x (\cos x - \sin x)$$

b. $y = t^2 \ln(\cos 2t)$

$$y' = 2t \ln(\cos 2t) + t^2 \cdot \frac{1}{\cos 2t} \cdot (-2 \sin 2t)$$

$$= 2t \ln(\cos 2t) - 2t^2 \tan 2t$$

c. $y = \int_2^{x^2} \frac{\sin t}{t^2} dt$

$$y' = \frac{\sin x^2}{x^4} \cdot 2x = \frac{2 \sin x^2}{x^3}$$

2. Integrate.

a. $\int t e^t dt$

$u=t \quad dv=e^t$
 $du=dt \quad v=e^t$

$$t e^t - \int e^t dt = t e^t - e^t + C$$

b. $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$

$u=\sqrt{x} \quad du=\frac{1}{2\sqrt{x}} dx$
 $2 du = \frac{1}{\sqrt{x}} dx$

$$\int 2 \cos u du = 2 \sin u + C = 2 \sin \sqrt{x} + C$$

c. $\int \frac{t}{1-t} dt$

$$= \int \frac{t}{t-1} dt = \int \frac{t-1+1}{t-1} dt = \int \left(1 + \frac{1}{t-1} \right) dt$$

$$= [t + \ln|t-1|] = t + \ln|t-1| + C$$

3. Write the following expressions in standard $a + bi$ form.

a. $e^{3+2i} = e^3 \cdot e^{2i}$

$$e^3 \cos 2 + i e^3 \sin 2$$

b. $t^{1-\sqrt{3}i}$

$$e^{(1-\sqrt{3}i) \ln t} = e^{\ln t} \cdot e^{-\sqrt{3}i \ln t}$$

$$t (\cos(\sqrt{3} \ln t) - i \sin(\sqrt{3} \ln t))$$

$$= t \cos(\sqrt{3} \ln t) - i t \sin(\sqrt{3} \ln t)$$

4. Determine whether the series $\sum_{n=1}^{\infty} \frac{n \ln n}{5^n}$ converges or diverges.

Converges

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{(n+1) \ln(n+1)}{5^{n+1}} \cdot \frac{5^n}{n \ln n} = \lim_{n \rightarrow \infty} \frac{(n+1) \ln(n+1)}{n \ln n} \cdot \lim_{n \rightarrow \infty} \frac{5^n}{5^n \cdot 5} = \frac{1}{5}$$

$$\frac{1}{5} < 1$$

5. Given $\vec{x} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\vec{y} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$, $A = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 1+3i & 2i \\ -6i & 4-i \end{pmatrix}$, find the following

a. $2\vec{x} - 3\vec{y}$

$$\begin{pmatrix} 4 \\ 6 \end{pmatrix} + \begin{pmatrix} +9 \\ -3 \end{pmatrix} = \begin{pmatrix} 13 \\ 3 \end{pmatrix}$$

b. $A + iB$

$$\begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix} + \begin{pmatrix} i-3 & -2 \\ 6 & 4i+1 \end{pmatrix} = \begin{pmatrix} -2+i & -3 \\ 8 & 5+4i \end{pmatrix}$$