MAT 212, Quiz #2, Spring 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Given $\vec{x} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}, A = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}, B = \begin{pmatrix} 1+3i & 2i \\ -6i & 4-i \end{pmatrix}$, find the following

   a. $A\vec{x}$

$$\begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} -3-1 \\ -6+4 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$$

   b. AB

$$\begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 1+3i & 2i \\ -6i & 4-i \end{pmatrix} = \begin{pmatrix} 1+3i+6i & 2i-4+i \\ 2+6i-24i & 4i+16-4i \end{pmatrix} = \begin{pmatrix} 1+9i & -4+3i \\ 2-18i & 16 \end{pmatrix}$$

   c. B^* (i.e. $\bar{B}^T$)

$$\bar{B} = \begin{pmatrix} 1-3i & -2i \\ 6i & 4+i \end{pmatrix} \quad B^* = \begin{pmatrix} 1-3i & 6i \\ -2i & 4+i \end{pmatrix}$$

2. Draw the direction field for $y' = y(y-3)$

$y(y-3)=0 \quad y=0, y=3$

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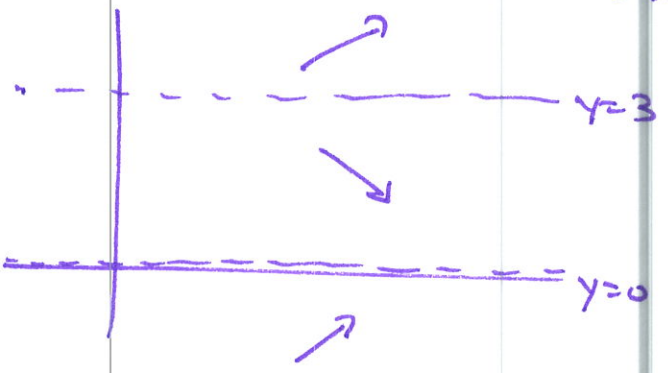

3. Solve for the particular solution to $\frac{dy}{dt} = y - 5, y(0) = 10$

$$\int \frac{dy}{y-5} = \int dt$$

$$\ln(y-5) = t + C$$

$$y-5 = e^{t+C} \Rightarrow y-5 = Ae^t$$

$$y(t) = 5 + Ae^t$$

$$y(0) = 5 + Ae^0 = 10 \Rightarrow A = 5$$

$$\boxed{y(t) = 5 + 5e^t}$$

4. Classify the following differential equations as i) ordinary or partial, ii) linear or nonlinear, iii) the order of the equation.

a. $u_x + uu_x = 1 + u_{yy}$

partial, nonlinear, 2nd order

b. $t^2y'' - 4ty' + 4y = 0$

ordinary, linear, 2nd order

c. $y^{IV} + 4y''' + 3y + t = 0$

ordinary, linear, 4th order

d. $\frac{d^2y}{dt^2} + \sin(t + y) = \sin t$

ordinary, nonlinear, 2nd order

e. $u_{xx} + u_{yy} = 0$

partial, linear, 2nd order

f. $(1 + y^2)\frac{dy}{dt} + y = e^t$

ordinary, nonlinear, 1st order