MAT 212, Quiz #3, Spring 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Use a matrix to row reduce the system $\begin{cases} 3x_1 + 4x_2 = 11 \\ x_1 + 5x_2 = 11 \end{cases}$ to solve for $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.

$\left[\begin{array}{cc|c} 3 & 4 & 11 \\ 1 & 5 & 11 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 5 & 11 \\ 3 & 4 & 11 \end{array}\right]$ $-3R_1 + R_2 \to R_2$ $\left[\begin{array}{cc|c} 1 & 5 & 11 \\ 0 & -11 & -22 \end{array}\right]$

$-3 \quad -15 \quad -33$

$-\frac{1}{11}R_2 \to R_2$ $\left[\begin{array}{cc|c} 1 & 5 & 11 \\ 0 & 1 & 2 \end{array}\right] \Rightarrow x_2 = 2$

$x_1 + 5x_2 = 11$

$x_1 + 5(2) = 11$

$x_1 + 10 = 11$

$x_1 = 1$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

2. Solve the linear ODE $2y' + y = 3t$ using an integrating factor.

$y' + \frac{1}{2}y = \frac{3}{2}t$ $\qquad \mu = e^{\int 1/2\, dt} = e^{1/2 t}$

$e^{1/2 t}y' + \frac{1}{2}e^{1/2 t}y = \frac{3}{2}te^{1/2 t}$

$du = t \quad dv = e^{1/2 t}$

$du = dt \quad v = 2e^{1/2 t}$

$\int (e^{1/2 t}y)' = \int \frac{3}{2}te^{1/2 t}\, dt$

$e^{1/2 t}y = \int \frac{3}{2}te^{1/2 t}\, dt = \frac{3}{2}\left[2te^{1/2 t} - \int 2e^{1/2 t}\, dt\right] = \frac{3}{2}\left[2te^{1/2 t} - 4e^{1/2 t} + C\right]$

$e^{1/2}y = 3te^{1/2 t} - 6e^{1/2 t} + C \Rightarrow \boxed{y = 3t - 6 + Ce^{-1/2 t}}$

3. A tank has pure water flowing into it at 10 L/min. The contents of the tank are kept thoroughly mixed, and the contents flow out at 10 L/min. Initially, the tank contains 10 kg of salt in 100 L of water. How much salt will there be in the tank after 30 minutes?

Rate in $= \frac{10L}{min} \cdot \frac{0 kg}{L} = 0$ $\qquad Q(0) = 10$ kg

Rate out $= \frac{10L}{min} \cdot \frac{Q kg}{100L} = \frac{Q}{10}$ $\qquad \frac{dQ}{dt} = $ Rate in $-$ Rate out

$\frac{dQ}{dt} = -\frac{Q}{10} \Rightarrow \int \frac{dQ}{Q} = \int -\frac{1}{10}dt \Rightarrow \ln Q = -\frac{1}{10}t + C$

$Q = e^{-\frac{1}{10}t + C} \Rightarrow Q(t) = Q_0 e^{-\frac{1}{10}t} \Rightarrow Q(t) = 10e^{-\frac{1}{10}t}$

$Q(30) = 10e^{-\frac{1}{10}\cdot 30} = 10e^{-3} \approx .49787$ kg

4. For the ODE $\frac{dy}{dt} = \frac{1+t^2}{3y-y^2}$, determine where a solution exists. Sketch the region in the plane. (Be sure to show explicitly that you check **both** conditions.)

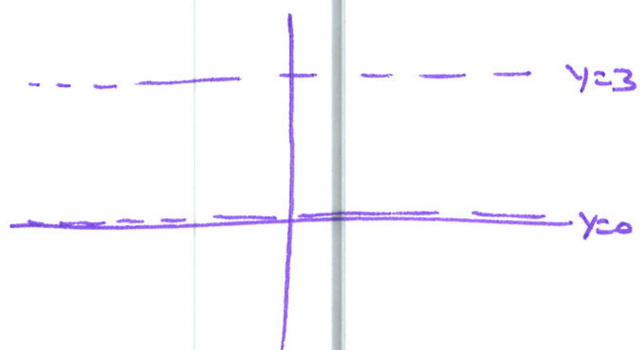
$3y - y^2 = y(3-y) = 0$

$y = 0,\ y = 3$

$\frac{dy}{dt}$ undefined when

$3y - y^2 = 0$

defined for all t



defined everywhere except these 2 values.

$y' = f(t,y) = (1+t^2)(3y-y^2)^{-1}$

$$\frac{\partial f}{\partial y} = (1+t^2)(-1)(3y-y^2)^{-2}(3-2y)$$

$$= \frac{(1+t^2)(2y-3)}{(3y-y^2)^2}$$

still undefined at $y=0,\ y=3$