MAT 100, Exam #1, Spring 2017 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each problem.

1. Find the first five terms of the sequence $A_N = \left(-\frac{1}{N}\right)^{N-1}$. (6 points)

$A_1 = \left(-\frac{1}{1}\right)^{1-1} = (-1)^0 = 1 \quad A_2 = \left(-\frac{1}{2}\right)^{2-1} = \left(-\frac{1}{2}\right)^1 = -\frac{1}{2}$

$A_3 = \left(-\frac{1}{3}\right)^{3-1} = \left(-\frac{1}{3}\right)^2 = \frac{1}{9}, \quad A_4 = \left(-\frac{1}{4}\right)^{4-1} = \left(-\frac{1}{4}\right)^3 = -\frac{1}{64}$

$A_5 = \left(-\frac{1}{5}\right)^{5-1} = \left(-\frac{1}{5}\right)^4 = \frac{1}{625}$

$1, -1/2, 1/9, -\frac{1}{64}, \frac{1}{625}, \ldots$

2. Find a formula for the sequence $\underbrace{21, 28}_{7=d}, 35, \ldots$ (6 points)

$A_N = 21 + 7N \; \}_{n=0}$ or $A_n = 21 + 7(N-1) = 14 + 7N \; \}_{n=1}$

3. Find the sum of $21 + 28 + 35 + \cdots + 413$. [Hint: first determine which term in the sequence is 413.] (8 points)

$413 = A_{N-1} = 21 + 7(N-1) \Rightarrow N-1 = 56 \Rightarrow N = 57$

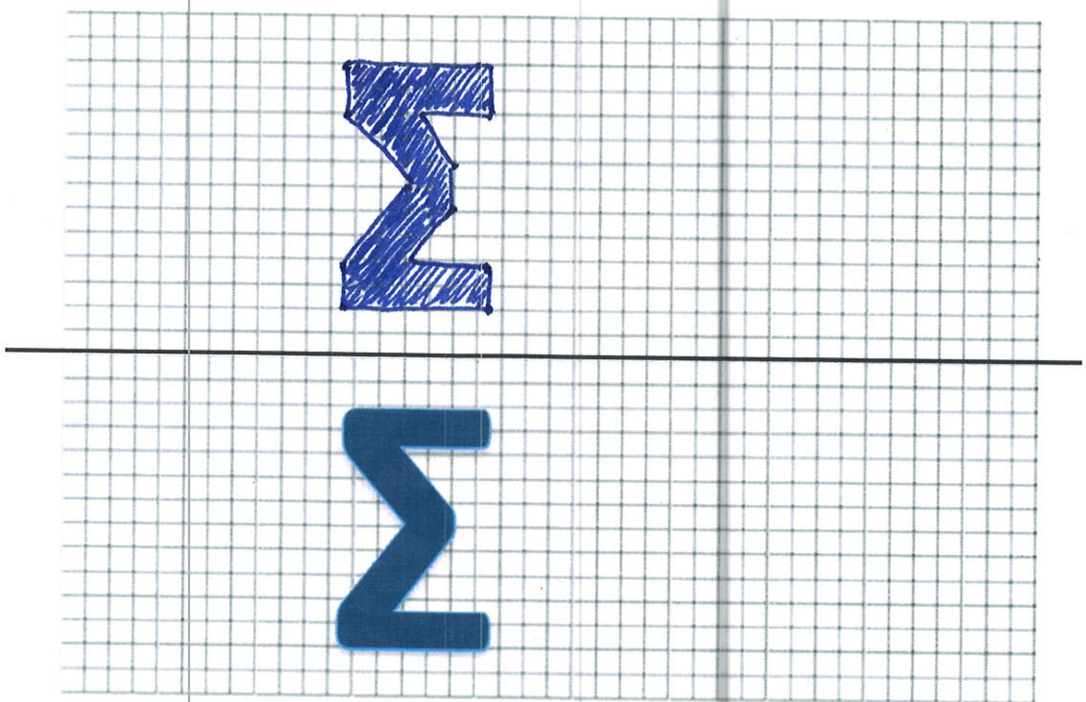
$\frac{(21+413)57}{2} = 12,369$

4. A geometric series is defined by $P_N = 4P_{N-1}$, $P_0 = \frac{1}{8}$. Find the sum of the first 20 terms. (7 points)

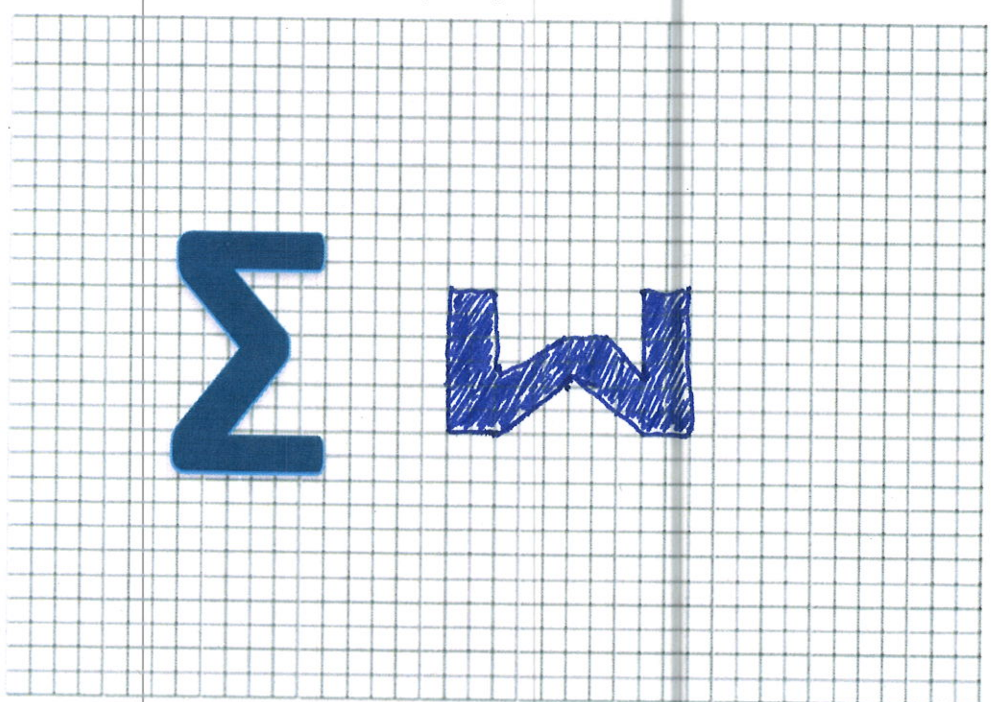
$P_0 + \ldots + P_{N-1} =$ P_{20} $R = 4$

$= \frac{\frac{1}{8}(1-4^{21})}{1-4} \approx 1.8325 \times 10^{11}$

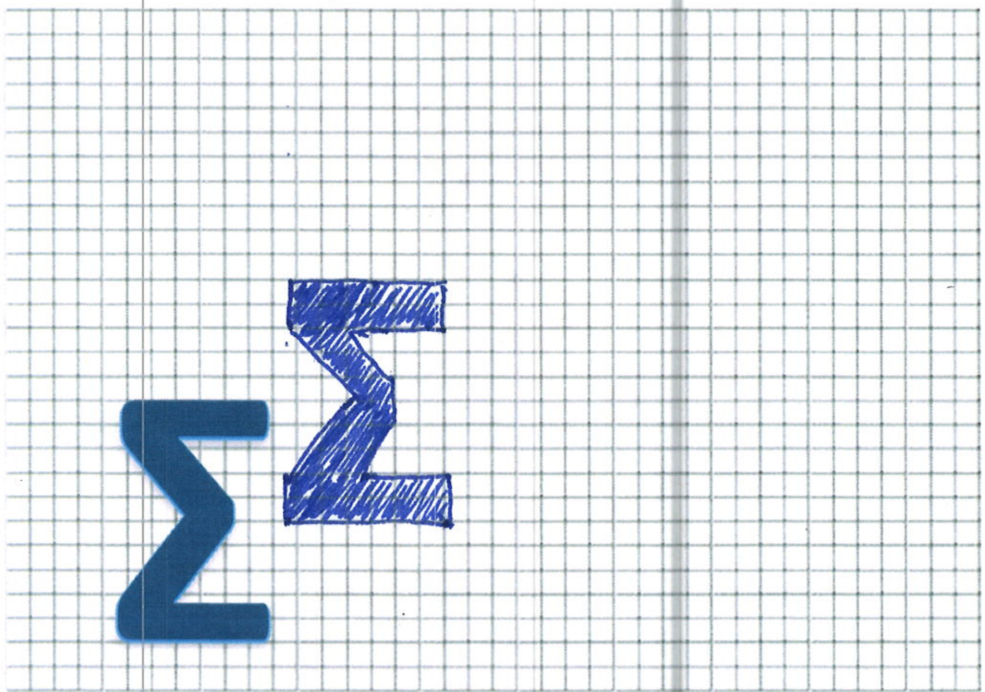
5. Perform the indicated rigid motions.
   a. Reflect across the indicated line. (7 points)



   b. Rotate by 90° counterclockwise. (7 points)



c. Translate by the vector $\vec{v} = \langle 7,5 \rangle$ (7 points)



6. State the symmetries of the border pattern below. [You do not need to provide notation.] (5 points)

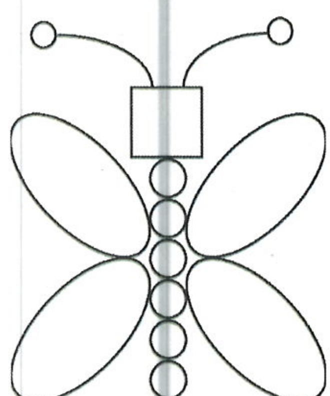
rotations - 180° & 360°

translation

no reflection symmetry

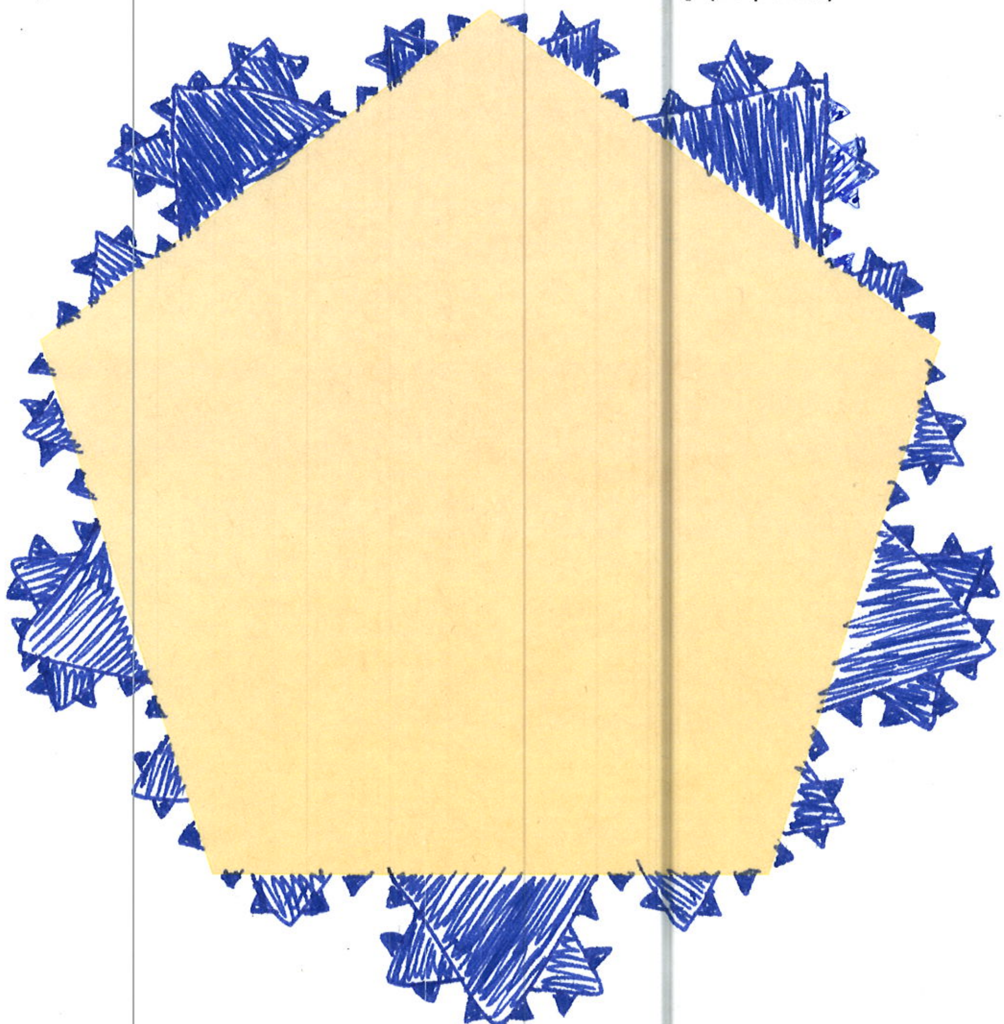
7. For each of the shapes below, use correct notation to indicate which type of symmetries are present. (5 points each).

D_7



D_1

8. Apply the Koch snowflake replacement rule to the sides of the pentagon shown below for three stages. [Recall: ] (10 points)



9. Define self-similarity. (6 points)

an object is self-similar if it remains the same (similar to itself) under magnification

10. Determine if $\frac{1}{3}+\frac{1}{4}i$ is inside the Mandelbrot set. Justify your conclusion with appropriate mathematics. (8 points)

$S_{N+1} = S_N^2 + (1/3 + 1/4 i)$

$S_1 = .38\ldots + .41\ldots i$

$S_2 = .305\ldots + .56\ldots i$

$S_3 = .10\ldots + .59\ldots i$

$S_4 = -.01\ldots + .3\ldots i$

$S_5 = .19\ldots + .24\ldots i$

$S_6 = .31\ldots + .34\ldots i$

$S_7 = .31\ldots + .46\ldots i$

$S_8 = .21\ldots + .54\ldots i$

$S_9 = .08\ldots + .48\ldots i$

$S_{10} = .10\ldots + .33\ldots i$

$S_{11} = .23\ldots + \ldots 32\ldots i$

$S_{12} = .28\ldots + .40\ldots i$

after many more steps seems to be staying near

$S_N = .20 + .41\ldots i$

.1998...

in Mandelbrot set

11. Find F_{17} and F_{18}. Then find the ratio, $\frac{F_{18}}{F_{17}}$. (8 points)

$F_{18} = 2584$

$F_{17} = 1597$

$\frac{2584}{1597} = 1.618033813$

12. Explain why the Golden Ratio is important enough to have its own symbol ϕ. (5 points)

answers may vary

it's irrational $= \frac{1+\sqrt{5}}{2}$

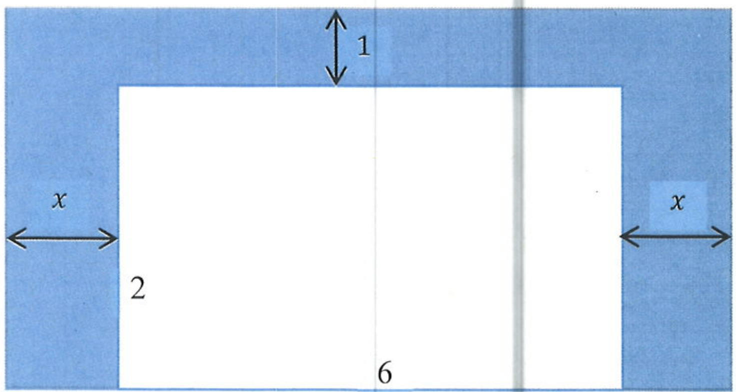
Fibonacci ratio approaches the value

it's the golden ratio

Satisfies the equation $\varphi^2 = \varphi + 1$

$\varphi - 1 = \frac{1}{\varphi}$

13. Find the value of x so that the shaded region is a gnomon of the smaller rectangle. (10 points)



$$\frac{6+2x}{6} = \frac{3}{2} \Rightarrow$$

$$2(6+2x) = 18$$

$$6+2x = 9$$

$$-6 \quad -6$$

$$\frac{2x}{\cancel{2}} = \frac{3}{2} \Rightarrow \boxed{x = \frac{3}{2}}$$

Some useful formulas:

$$P_N = P_0 + Nd$$

$$\sum_{i=0}^{N} P_i = \frac{(P_0 + P_{N-1})N}{2}$$

$$P_N = P_0 R^N$$

$$\sum_{i=0}^{N} P_i = P_0 \left(\frac{1 - R^N}{1 - R} \right)$$

$$S_{N+1} = (S_N)^2 + s$$

$$F_N = \left[\!\left[\left(\frac{1+\sqrt{5}}{2} \right)^N / \sqrt{5} \right]\!\right]$$

$$\phi^2 = \phi + 1$$