MAT 100, Final Exam, Spring 2017 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each problem.

1. You buy a TV for $2000 at a simple interest of 6% for 2 years. How much money will you need to pay back at the end of the loan? (8 points)

$2000(1+.06(2)) = \$2240$

2. You place $10,000 in a CD paying 2.1% interest annually. If interest is compounded weekly, how much money is in the account 5 years later? (8 points)

$10{,}000\left(1+\frac{.021}{52}\right)^{5.52} =$

$\$11{,}106.87$

N = 260
I = 2.1
PV = 10,000
PMT = 0
FV = 11,106.87
P/Y = C/Y = 52

3. Suppose you invest your $100,000 retirement account balance in the stock market for ten years, earning an average return of 9% compounded continuously. How much money is in the account at the end of that time? (8 points)

$100{,}000e^{.09(10)} = \$245{,}960.31$

4. Find the effective rate for 3.6% compounded
   a. Daily (4 points)

$$1-\left(1+\frac{.036}{365}\right)^{365} = 1-1.03665...$$

3.67%

   b. Semi-monthly (4 points)

$$1-\left(1+\frac{.036}{24}\right)^{24} = 1-1.0366627...$$

3.66%

5. If you take out a mortgage of $250,000 with a 30-year term, at 2.8% interest,
   a. How much are your monthly payments? (8 points)

N = 360
I = 2.8
PV = 250,000
PMT = 1027.24
FV = 0
P/Y = Y/Y = 12

$1027.24

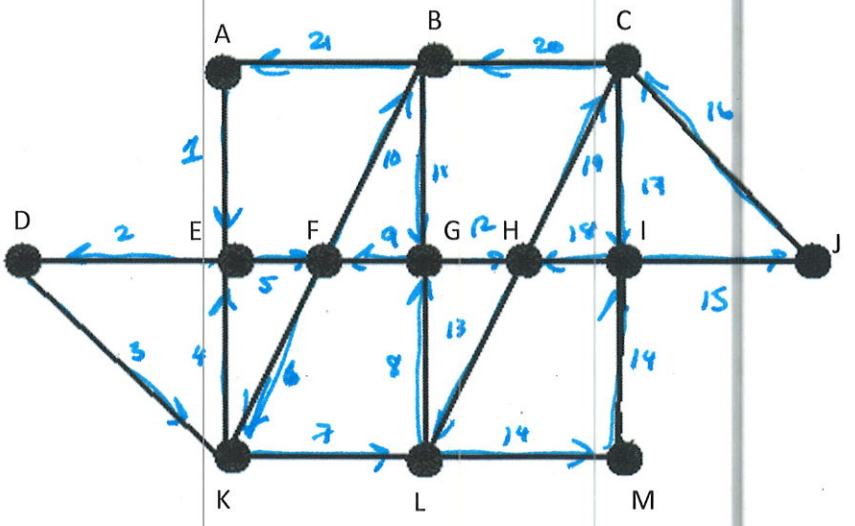
   b. How much interest do you pay back by the time the loan is paid off? (5 points)

$1027.24 * 360 = 369,806.40

$$-\ 250,000.00$$

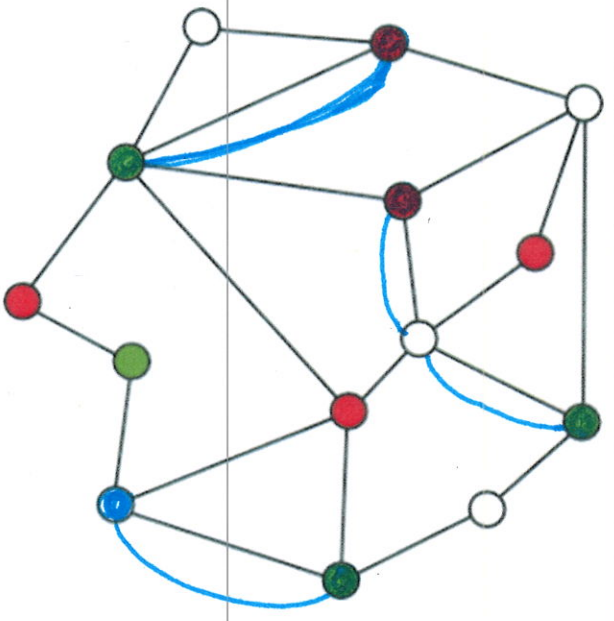
$119,806.40

6. Use Fleury's Algorithm on the graph below to find an Euler circuit (or path). List the vertices you travel through in order, or number the edges as you use them. (8 points)



answers will vary

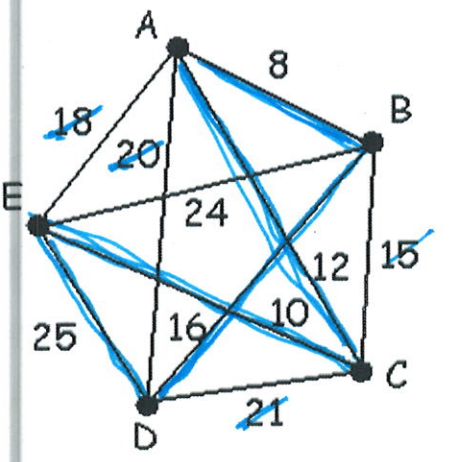
7. Eulerize the graph below. What is the minimum number of edges that might be used to eulerize this graph, and how many did you actually use? (8 points)



6 odd vertices need minimum of 3 to Eulerize

actually need 4

8. Use the graph to the right to find the (approx.) minimum cost Hamilton circuit by the Cheapest Link Algorithm. Clearly state the final weight of your circuit. (10 points)



8+10+12+16+25 = 71

9. Use the Nearest Neighbor Algorithm to find the (approx.) lowest cost Hamilton circuit using the table below. What is the length of the final circuit? (10 points)

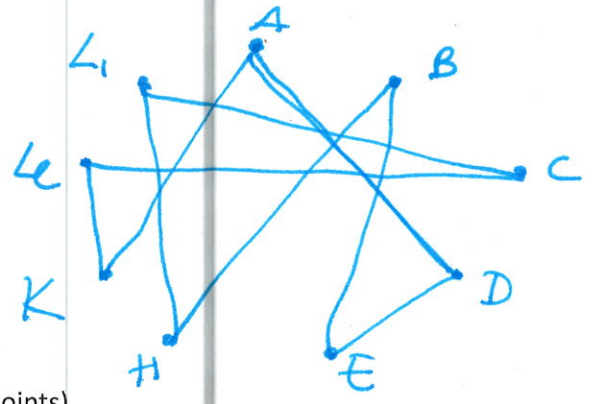
Aberdeen								
513	Bristol							
473	171	Cambridge						
595	206	124	Dover					
587	83	250	244	Exeter				
482	54	153	224	128	Hereford			
279	236	252	355	310	204	Kendal		
328	219	147	272	294	188	72	Leeds	
388	185	94	219	259	153	177	142	Lincoln

Distance in kilometres

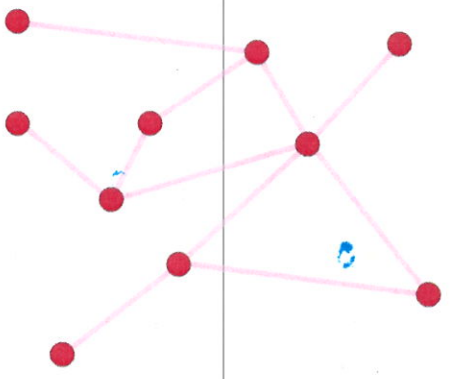
Starting Aberdeen

A K Le C Li H B E D A

279 + 72 + 147 + 94 + 153 + 54 + 83 + 244 + 595 = 1721



10. Find the redundancy of the graph below. (5 points)



10 vertices

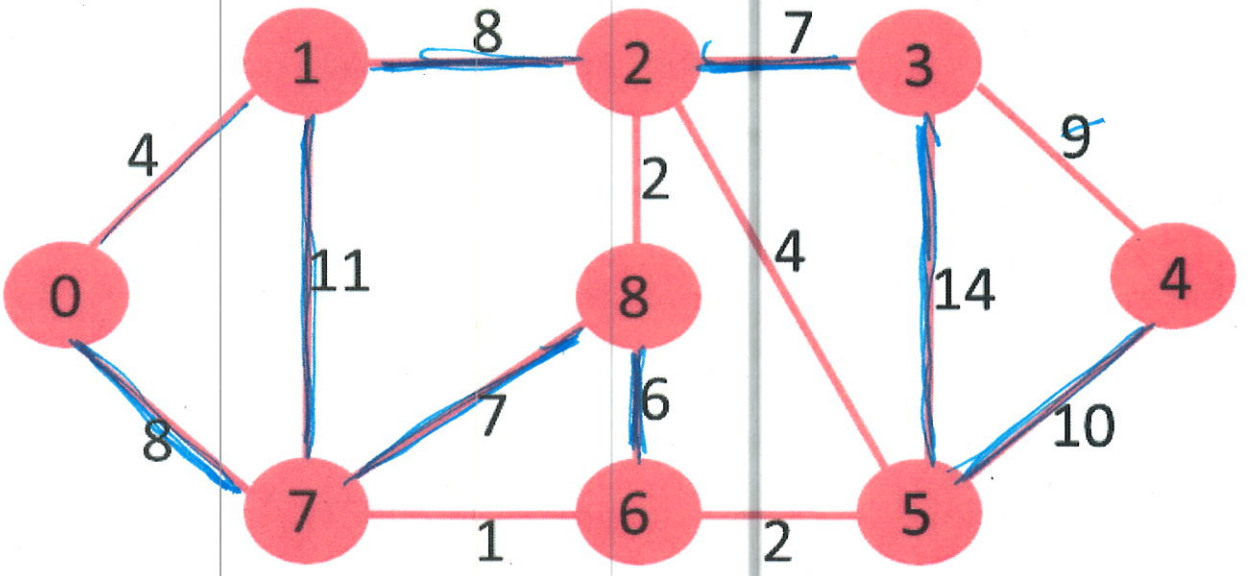
9 needed for tree (edges)

11 edges in graph (2 circuits)

11 − 9 = 2

redundancy is 2

11. Use Kruskal's Algorithm to find the highest value minimum spanning tree. Be sure to state the weight of the final tree. (10 points)



8+11+8+7+6+7+14+10 = 71

12. Use the following preference schedule to find the winner of the election using the indicated method.

18	12	10	9	4	2
Allen	~~Evert~~	Davis	~~Baker~~	~~Childs~~	~~Childs~~
~~Baker~~	Childs	~~Evert~~	Davis	~~Evert~~	Davis
~~Childs~~	~~Baker~~	~~Childs~~	~~Childs~~	~~Baker~~	~~Baker~~
Davis	Davis	~~Baker~~	~~Evert~~	Davis	~~Evert~~
~~Evert~~	Allen	Allen	Allen	Allen	Allen

a. Borda Count Method (10 points)

A: 18×5 + 12×4 + 10×4 + 9×1 + 4×1 + 2×1 = 127
B: 18×4 + 10×3 + 9×5 + 12×3 + 4×3 + 2×3 = 191 ←
C: 18×3 + 12×4 + 10×3 + 9×3 + 4×5 + 2×5 = 189
D: 18×2 + 12×2 + 10×5 + 9×4 + 4×2 + 2×4 = 162
E: 18×1 + 12×5 + 10×4 + 9×2 + 4×4 + 2×2 = 138

B wins

b. Plurality with Elimination Method (8 points)

Rnd 1
A: 18
B: 9
~~C: 6~~
D: 10
E: 12

Rnd 2
A: 18
~~B: 9~~
D: 12
E: 16

Rnd 3
A: 18
D: 21
~~E: 16~~

Rnd 4
A: 18
D: 37

D wins

13. A County Elections Board needs to allocate 50 "floating" pollworkers to various communities during an election to troubleshoot problems that arise. The communities and their registered voting populations are noted in the table below. Use the tables to apportion the pollworkers to the various communities.

Town	Population	Standard Quota	Lower Quota	Upper Quota	Extra Seat?	Final Apportionment
Oakcastle	4139	10.866	10	11	+1	11
Southhaven	2509	6.587	6	7	+1	7
Whitefaire	5267	13.828	13	14	+1	14
Easthill	1311	3.442	3	4		3
Eriden	5819	15.277	15	16		15
Standard Divisor =		380.9	47			50

total pop 19045

a. By Hamilton's Method (10 points)

Town	Population	Standard Quota	Lower Quota	Upper Quota	Geometric Mean	Final Apportionment
Oakcastle	4139	10.866	10	11	10.488	11
Southhaven	2509	6.587	6	7	6.481	7
Whitefaire	5267	13.828	13	14	13.491	14
Easthill	1311	3.442	3	4	3.464	3
Eriden	5819	15.277	15	16	15.492	15
Standard Divisor =		380.9				50

b. By Huntington-Hill's Method. (10 points)

14. The table of 20 math test scores is shown below. Find the indicated statistics.

78	75	76	71	88	80	74	70	78	75
65	81	84	72	76	68	79	73	44	99

a. Mean and mode (if it exists). (4 points)

$\bar{X} = 75.3$ mean / no mode

b. The 5-number summary (5 points)

min = 44 $Q_3 = 79.5$

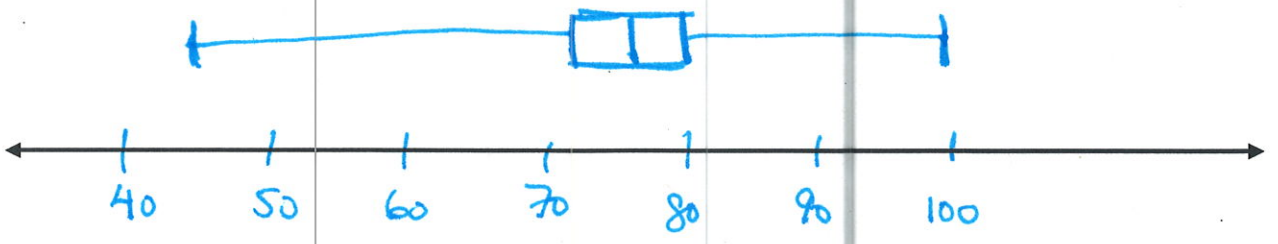
$Q_1 = 71.5$ Max = 99

Med = 75.5

c. (Sample) standard deviation. (4 points)

$S = 10.48$

d. Draw a box plot of the data (to scale). (6 points)



15. A card is drawn from a standard deck.

a. What is the probability the first card drawn is a jack? (3 points)

$\frac{4}{52} = \frac{1}{13}$

b. What is the probability of drawing a spade, and then another spade (without replacement)? (4 points)

$\frac{13}{52} \cdot \frac{12}{51} = \frac{1}{17}$

16. How many outcomes are possible in the following scenarios?

a. A password is 12 characters long, is case sensitive and numbers can be used. How many such passwords are possible? (5 points)

$62^{12} = 3.23 \times 10^{21}$

b. A board of directors has 17 members. How many ways can they elect a slate of four officers (president, vice president, treasurer, and secretary)? (5 points)

$17P4 = 57,120$

c. The math department selects 5 faculty members to serve on a special committee. How many different committees are possible if there are 28 faculty in the department? (5 points)

$28C5 = 98,280$

17. A charity raffle sells 375 tickets at a cost of \$8 each. They give away the following prizes: 1 first-place prize worth \$2500, 1 second-place prize worth \$700, 2 third-place prizes worth \$150, and 5 fourth-place prizes worth \$20. Find the expected value of purchasing a ticket. (8 points)

x	2492	692	142	12	-8
$p(x)$	$1/375$	$1/375$	$2/375$	$5/375$	$366/375$

$2492 \cdot 1/375 + 692(1/375) + 142(2/375) + 12(9/375) - 8(366/375) =$

$-\$1.6$

18. A geometric series is defined by $P_N = 3P_{N-1}$, $P_0 = 8$. Find the sum of the first 50 terms. (6 points)

$$\frac{8(1-3^{51})}{1-3} = 8.61 \times 10^{24}$$

19. Find a formula for the sequence $12, 21, 30, 39, \ldots$ Then use the formula to find P_{25}. (8 points)

Starting at $n=0$ $a_0 = 12$ $d=9$

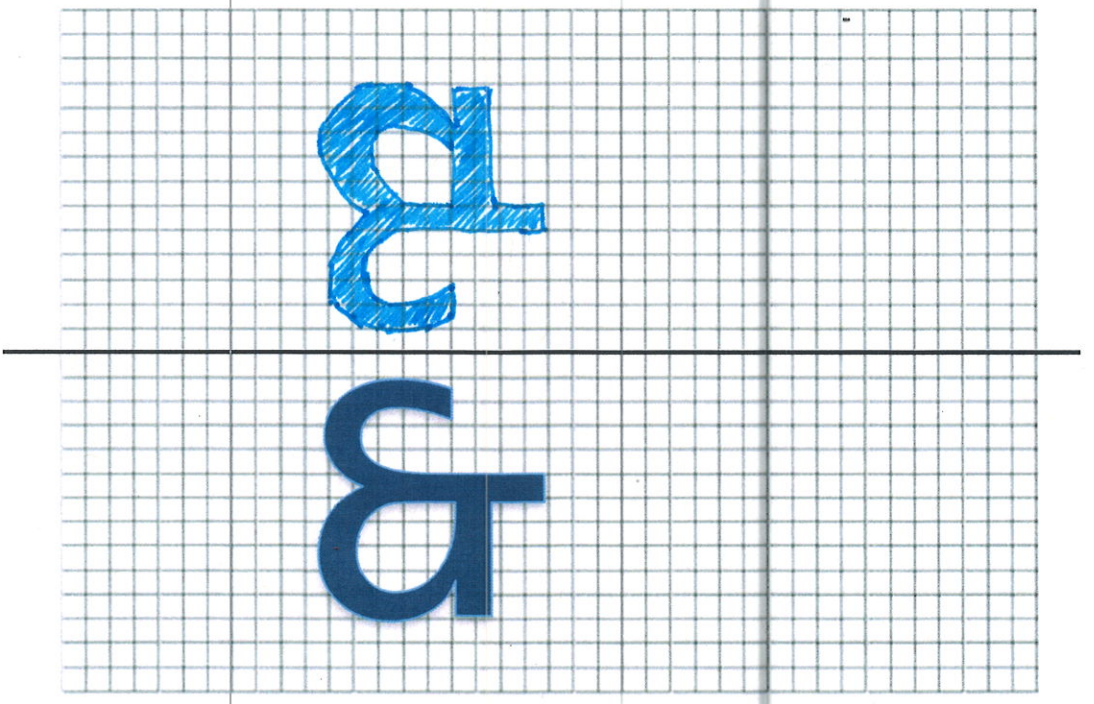
$a_n = 12+9n$ $a_{25} = 12+9(25) = 237$

Starting at $n=1$ $a_1 = 12$ $d=9$

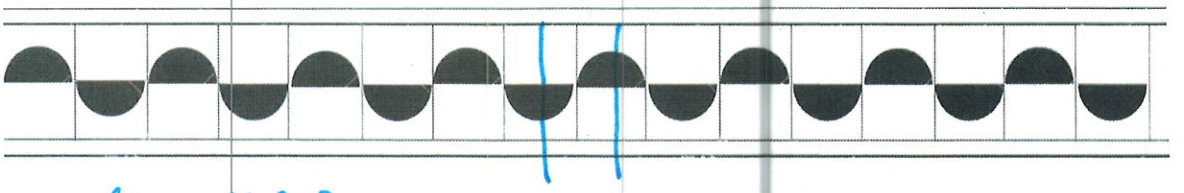
$a_n = 12+9(n-1) = 3+9n$

$a_{25} = 3+9(25) = 228$

20. Reflect across the indicated line. (8 points)



21. State the symmetries of the border pattern below. [You do not need to provide notation.] (6 points)



rotation 180°, 360°
glide reflection
translation

2 reflections

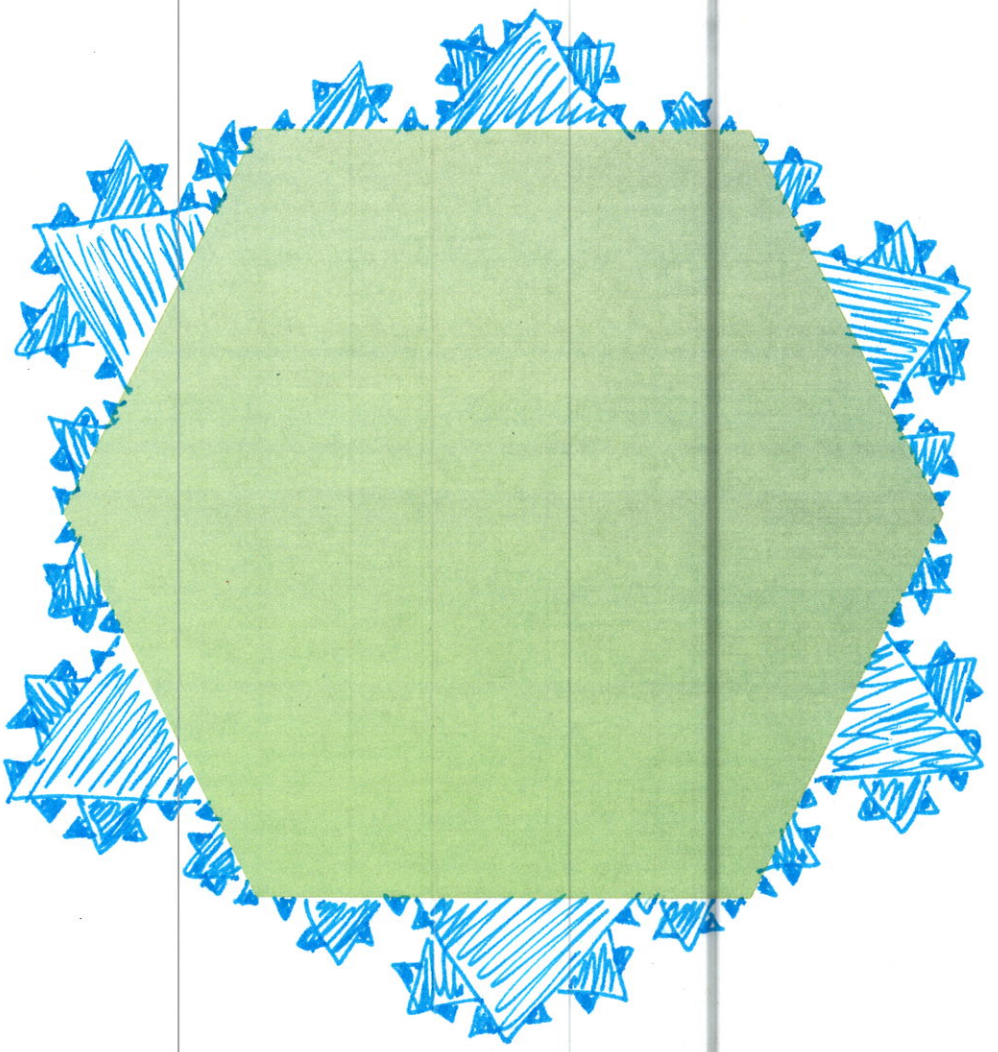
22. Find F_{12} and F_{11}. Then find the ratio, $\frac{F_{12}}{F_{11}}$. (8 points)

$F_{11} = 89$

$F_{12} = 144$

$\frac{144}{89} = 1.617977528$

23. Apply the Koch snowflake replacement rule to the sides of the pentagon shown below for three stages. [Recall: ] (10 points)



Some useful formulas:

$$FV = P(1 + rt)$$
$$I = Prt$$

$$FV = P(1 + r)^T$$

$$FV = Pe^{rt}$$

$$M = \frac{Pp(1 + p)^T}{[(1 + p)^T - 1]}$$

$$s = \sqrt{\sum \frac{(x_i - \bar{x})^2}{n - 1}}$$

$$P_N = P_0 + Nd$$
$$\sum_{i=0}^{N} P_i = \frac{(P_0 + P_{N-1})N}{2}$$

$$P_N = P_0 R^N$$
$$\sum_{i=0}^{N} P_i = P_0 \left(\frac{1 - R^N}{1 - R}\right)$$

$$S_{N+1} = (S_N)^2 + s$$

$$F_N = \left[\left[\left(\frac{1 + \sqrt{5}}{2}\right)^N / \sqrt{5}\right]\right]$$

$$\phi^2 = \phi + 1$$