MAT 142, Exam #1, Spring 2017 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round. Complete all parts of each question. Questions that provide only answers and no work will not receive full credit. If you use your calculator (only when problems don't instruct you to do the problem by hand), showing calculator steps will count as "work".

1. Simplify. $\dfrac{\frac{3}{x-2}-\frac{4}{x+2}}{\frac{7}{x^2-4}}$ $\dfrac{(x+2)(x-2)}{(x+2)(x-2)}$ (5 points)

$(x-2)(x+2)$

$$\frac{3(x+2)-4(x-2)}{7} = \frac{3x+6-4x+8}{7} = \boxed{\frac{-x+14}{7}}$$

2. Solve.
   a. $\left[5+\frac{x-2}{3}=\frac{x+3}{8}\right]24$ (4 points)

$120+8(x-2)=3(x+3)$

$120+8x-16=3x+9$

$104+8x=3x+9$

$-104 \qquad -104$

$8x=3x-95$

$-3x \quad -3x$

$\frac{5x}{5}=\frac{-95}{5}$

$\boxed{x=-19}$

   b. $\frac{4}{x^2+3x-10}-\frac{1}{x^2+x-6}=\frac{3}{x^2-x-12}$ (5 points)

$(x+5)(x-2) \quad (x+3)(x-2) \quad (x-4)(x+3)$

$LCD=(x+5)(x-2)(x-4)(x+3)$

$4(x-4)(x+3)-1(x+5)(x-4)=3(x+5)(x-2)$

$4(x^2-x-12)-(x^2+x-20)=3(x^2+3x-10)$

$4x^2-4x-48-x^2-x+20=3x^2+9x-30$

$\cancel{3x^2}-5x-28=\cancel{3x^2}+9x-30$

$-9x+28 \qquad -9x+28$

$\frac{-14x}{-14}=\frac{-2}{-14}$

$\boxed{x=\frac{1}{7}}$ does not make denominator 0 so, its okay!

3. Solve. Find all real or complex solutions.

a. $x^2 + 4x + 1 = 0$ (4 points)

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)}}{2} = \frac{-4 \pm \sqrt{12}}{2} = \frac{-4 \pm 2\sqrt{3}}{2} = \boxed{-2 \pm \sqrt{3}}$$

b. $2x^2 - 7x + 3 = 0$ (4 points)

$$(2x-1)(x-3) = 0$$

$$\boxed{x = 1/2,\ x = 3}$$

c. $(2x-5)(x+1) = 2$ (5 points)

$$2x^2 + 2x - 5x - 5 = 2$$
$$-2$$
$$2x^2 - 3x - 7 = 0$$

$$x = \frac{3 \pm \sqrt{9 + 56}}{4} = \boxed{\frac{3 \pm \sqrt{65}}{4}}$$

d. $\sqrt{(2x+8)^2} = \sqrt{27}$ (4 points)

$$2x + 8 = \pm\sqrt{27} = \pm 3\sqrt{3}$$
$$-8 \qquad -8$$

$$\frac{2x}{2} = \frac{\pm 3\sqrt{3} - 8}{2}$$

$$\boxed{x = -4 \pm \tfrac{3}{2}\sqrt{3}}$$

4. Solve.

a. $\left(\sqrt{x+10}\right)^2 = (x-2)^2$ (6 points)

$$x + 10 = x^2 - 4x + 4$$
$$-x - 10 \qquad -x - 10$$

$$0 = x^2 - 5x - 6$$
$$(x-6)(x+1) = 0$$
$$x = 6,\ x = -1$$

Check!

$\sqrt{6+10} = \sqrt{16} = 4\ ?\ 6 - 2 = 4$ ✓

$\sqrt{-1+10} = \sqrt{9} = 3\ ?\ -1 - 2 = -3$ ✗ no

$$\boxed{x = 6}$$

b. $6x^{5/2} - 12 = 0$ (4 points)

$\frac{6x^{5/2}}{6} = \frac{12}{6}$

$x^{5/2} = 2$

$x = 2^{2/5} = \sqrt[5]{4}$

c. $\left(y - \frac{8}{y}\right)^2 + 5\left(y - \frac{8}{y}\right) - 14 = 0$ (6 points)

$u = y - \frac{8}{y}$

$u^2 + 5u - 14 = 0$

$(u+7)(u-2) = 0$

$u = -7, u = 2$

$\left(y - \frac{8}{y} = -7\right)y$

$y^2 - 8 = -7y \Rightarrow y^2 + 7y - 8 = 0$

$(y+8)(y-1) = 0$ $\boxed{y = -8, y = 1}$

$\left(y - \frac{8}{y} = 2\right)y$

$y^2 - 8 = 2y \Rightarrow y^2 - 2y - 8 = 0$

$(y-4)(y+2) = 0$ $\boxed{y = 4, y = -2}$

d. $2\left|4 - \frac{5}{2}x\right| + 6 = 18$ (4 points)

$-6 \quad -6$

$\frac{2\left|4 - \frac{5}{2}x\right|}{2} = \frac{12}{2}$

$\left|4 - \frac{5}{2}x\right| = 6$

$4 - \frac{5}{2}x = 6$

$-4 \quad -4$

$\left(-\frac{2}{5}\right) - \frac{5}{2}x = 2\left(-\frac{2}{5}\right)$

$\boxed{x = -\frac{4}{5}}$

$4 - \frac{5}{2}x = -6$

$-4 \quad -4$

$\left(-\frac{2}{5}\right) - \frac{5}{2}x = -10\left(-\frac{2}{5}\right)$

$\boxed{x = 4}$

5. Factor completely. (3 points each)

a. $2x^4 - 162$

$2(x^4 - 81) = 2(x^2-9)(x^2+9) = 2(x-3)(x+3)(x^2+9)$

b. $x^3 - 125$

$(x-5)(x^2+5x+25)$

c. $(x+5)^{-3/2} - (x+5)^{-9/2}$

$\frac{1}{(x+5)^{3/2}} - \frac{1}{(x+5)^{9/2}} = \frac{(x+5)^3}{(x+5)^{9/2}} - \frac{1}{(x+5)^{9/2}} =$

$\frac{x^3 + 15x^2 + 75x + 125 - 1}{(x+5)^{9/2}} = (x+5)^{-9/2}\left[x^3 + 15x^2 + 75 + 124\right]$

d. $x^4 + 81$

Prime
Sum of squares

6. Find the indicated sum. (4 points each)

a. $\sum_{i=1}^{6} 7i$

$7(1) + 7(2) + 7(3) + 7(4) + 7(5) + 7(6) =$
$7[1+2+3+4+5+6] = 7(21) = \boxed{147}$

b. $\sum_{i=1}^{5} \frac{(i+2)!}{i!}$

$\frac{3!}{1!} + \frac{4!}{2!} + \frac{5!}{3!} + \frac{6!}{4!} + \frac{7!}{5!} =$

$\frac{6}{1} + \frac{24}{2} + \frac{120}{6} + \frac{720}{24} + \frac{5040}{120} = 6 + 12 + 20 + 30 + 42$

$= \boxed{110}$

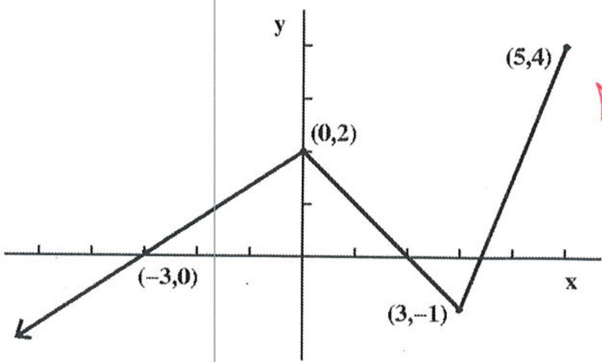
7. Write a formula in summation notation for $\frac{1}{3} + \frac{2}{4} + \frac{3}{5} + \cdots + \frac{16}{18}$. (3 points)

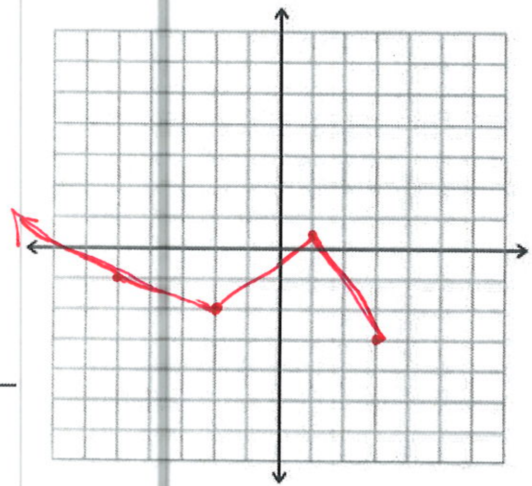
$\sum_{i=1}^{16} \frac{n}{n+2}$

8. Write a formula for the infinite series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots$, then find the sum. (5 points)

$\sum_{i=0}^{\infty} \left(-\frac{1}{2}\right)^i = \frac{1}{1-\left(-\frac{1}{2}\right)} = \frac{1}{1+\frac{1}{2}} = \frac{1}{\frac{3}{2}} = \boxed{\frac{2}{3}}$

9. Using the graph of $f(x)$ below, apply the transformation $-\frac{1}{2}f(x+2)-1$, redraw the result on the graph provided. (7 points)





Shift left 2 (x-value)

$-\frac{1}{2}$ * y value then subtract 1

$(0,2) \to (-2, -\frac{1}{2}(2)-1) \Rightarrow (-2,-2)$

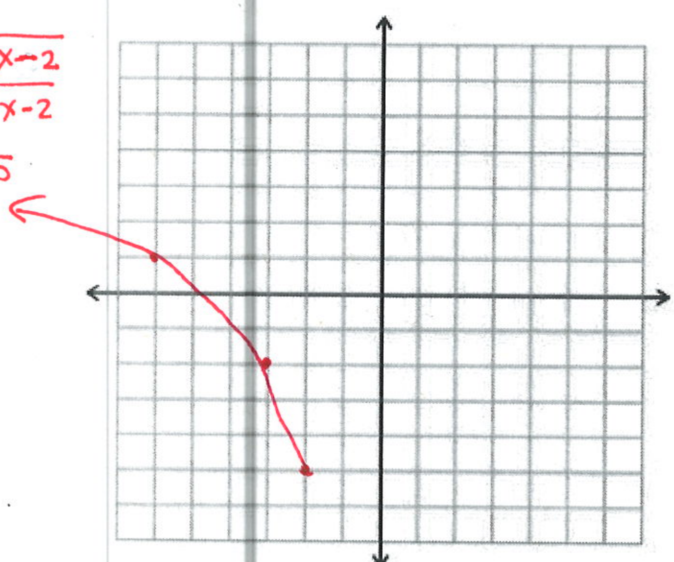
$(-3,0) \to (-5, -\frac{1}{2}(0)-1) \Rightarrow (-5,-1)$

$(3,1) \to (1, -\frac{1}{2}(1)-1) \Rightarrow (1, -\frac{1}{2})$

$(5,4) \to (3, -\frac{1}{2}(4)-1) \to (3,-3)$

10. Write the equation of the graph of $y=\sqrt{x}$ after the following transformations have been applied. Then graph the resulting equation. (7 points)
 a. Horizontal reflection $y=\sqrt{-x}$
 b. Horizontal shift left by two $y=\sqrt{-x-2}$
 c. Vertical stretch by three $y=3\sqrt{-x-2}$
 d. Vertical shift down by five

$y=3\sqrt{-x-2}-5$



11. For $f(x)=6-\frac{1}{x}$, and $g(x)=\frac{7}{x+8}$, find and simplify each of the following functions. State the domain of each. (4 points each)
 a. $f-g$

$$6-\frac{1}{x}-\frac{7}{x+8}=\frac{(6x-1)(x+8)-7x}{x(x+8)}=\frac{6x^2+48x-x-8-7x}{x(x+8)}$$

$$=\boxed{\frac{6x^2+40x-8}{x(x+8)}}$$

$x \neq 0, -8$

b. $\frac{f}{g}$

$$\frac{6-\frac{1}{x}}{\frac{7}{x+8}} \cdot \frac{x(x+8)}{x(x+8)} = \frac{6x(x+8)-(x+8)}{7x} =$$

$$\frac{6x^2+48x-x-8}{7x} = \boxed{\frac{6x^2+47x-8}{7x}}$$

$x \neq 0, -8$

c. $g \circ f$

$$\frac{7}{(6-\frac{1}{x})+8} = \frac{7}{6-\frac{1}{x}+8} = \frac{7}{14-\frac{1}{x}} \cdot \frac{x}{x} = \boxed{\frac{7x}{14x-1}}$$

$x \neq 0, \frac{1}{14}$

12. Find the inverse of $f(x) = \frac{2x+1}{x-3}$. State the domain of the inverse. (5 points)

$$(y-3)\; x = \frac{2y+1}{y-3}\;(y-3) \Rightarrow xy-3x = 2y+1$$

$$xy-2y = 3x+1$$

$$y(x-2) = 3x+1$$

$$y = \frac{3x+1}{x-2} = f^{-1}(x)$$

domain of $f^{-1}(x)$ is $(-\infty,2)\cup(2,\infty)$

all reals $\neq 2$

13. Find a composition of functions $f(x), g(x)$ so that $h(x) = \sqrt[3]{x^2-3}$ is the composition $f(g(x))$. (4 points)

answers may vary but

$f(x) = \sqrt[3]{x}$

$g(x) = x^2-3$

is one way.

14. Find and simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$ for $f(x) = -x^2 + 2x + 4$. (7 points)

$$\frac{-(x+h)^2+2(x+h)+4-(-x^2+2x+4)}{h} = \frac{\cancel{-x^2}-2xh-h^2+\cancel{2x}+2h+\cancel{4}+\cancel{x^2}-\cancel{2x}-\cancel{4}}{h}$$

$$= \frac{-2xh-h^2+2h}{h} = \frac{\cancel{h}(-2x-h+2)}{\cancel{h}} = \boxed{-2x-h+2}$$

15. Use long division to divide the polynomials $\frac{x^4+2x^3-4x^2-5x-6}{x^2+x-2}$. Write the result at $q(x) + \frac{r(x)}{d(x)}$. (7 points)

$$\begin{array}{r} x^2+x-3 \\ x^2+x-2 \overline{\smash{)}\, x^4+2x^3-4x^2-5x-6} \\ -\; x^4 \mp x^3 \pm 2x^2 \\ \hline x^3-2x^2-5x-6 \\ -\; x^3 \mp x^2 \pm 2x \\ \hline -3x^2-3x-6 \\ +3x^2+3x \mp 6 \\ \hline -12 \end{array}$$

$$\boxed{x^2+x-3-\frac{12}{x^2+x-2}}$$

16. Use synthetic division to divide $\frac{x^7+x^5-10x^3+12}{x+2}$. Write the result at $q(x) + \frac{r(x)}{d(x)}$. (6 points)

$$\begin{array}{r|rrrrrrrr} -2 & 1 & 0 & 1 & 0 & -10 & 0 & 0 & 12 \\ & & -2 & 4 & -10 & 20 & -20 & 40 & -80 \\ \hline & 1 & -2 & 5 & -10 & 10 & -20 & 40 & -68 \end{array}$$

$$x^6-2x^5+5x^4-10x^3+10x^2-20x+40-\frac{68}{x+2}$$

17. Use synthetic division and the Remainder Theorem to evaluate $f(x) = 4x^3 + 5x^2 - 6x - 4$ at $f(-2)$. (4 points)

$$\begin{array}{r|rrrr} -2 & 4 & 5 & -6 & -4 \\ & & -8 & 6 & 0 \\ \hline & 4 & -3 & 0 & -4 \end{array}$$

$f(-2) = -4$

18. Find all the rational zero of $f(x) = 2x^3 + x^2 - 3x + 1$ and use them to factor the polynomial completely to find all real and complex zeros. (8 points)

possible $\pm 1, \pm \frac{1}{2}$

$$\begin{array}{r|rrrr} 1 & 2 & 1 & -3 & 1 \\ & & 2 & 3 & 0 \\ \hline & 2 & 3 & 0 & 1 \end{array}$$

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & 1 & -3 & 1 \\ & & 1 & 1 & -1 \\ \hline & 2 & 2 & -2 & 0 \end{array}$$

rational zero $= \frac{1}{2}$
factor $2x-1$

$$\begin{array}{r|rrrr} -1 & 2 & 1 & -3 & 1 \\ & & -2 & 1 & 2 \\ \hline & 2 & -1 & -2 & 3 \end{array}$$

$$\begin{array}{r} x^2 + x - 1 \\ 2x-1 \overline{\smash{)}\, 2x^3 + x^2 - 3x + 1} \\ -\,2x^3 + x^2 \\ \hline 2x^2 - 3x + 1 \\ -\,2x^2 + x \\ \hline -2x + 1 \\ +\,2x \mp 1 \\ \hline 0 \end{array}$$

$x = \dfrac{-1 \pm \sqrt{1 - 4(-1)}}{2} = \dfrac{-1 \pm \sqrt{5}}{2}$

$\boxed{\text{Zeros: } \frac{1}{2}, \frac{-1 \pm \sqrt{\;}}{\;}}$

19. Find a fourth degree polynomial with zeros $-2, 5, 3 + 2i$, and $f(1) = -96$. (5 points)

$(x+2)(x-5)(x-3-2i)(x-3+2i)$

$(x+2)(x-5) = x^2 - 3x - 10$

$(x-3+2i)(x-3-2i) = x^2 - 3x \cancel{-2xi} - 3x + 9 \cancel{+6i} \cancel{+2xi} \cancel{-6i} + 4$

$x^2 - 6x + 13$

$(x^2 - 6x + 13)(x^2 - 3x - 10) = x^4 - 3x^3 - 10x^2 - 6x^3 + 18x^2 + 60x + 13x^2 - 39x - 130$

$= \boxed{(x^4 - 9x^3 + 21x^2 + 21x - 130)a \quad a = 1}$

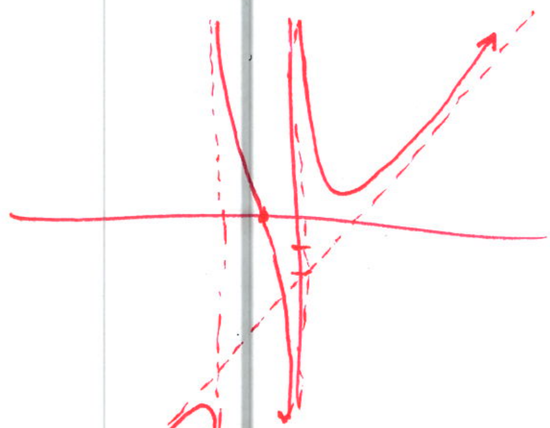
20. Find all vertical, horizontal or slant/oblique asymptotes, and any holes. Use that information to graph the function $f(x) = \frac{x^3+1}{x^2+2x}$. (8 points)

$$\begin{array}{r} x-2 \\ x^2+2x \overline{)\, x^3+0x^2+0x+1} \\ -x^3 \mp 2x^2 \\ \hline -2x^2+0x+1 \\ +2x^2+4x \\ \hline 4x+1 \end{array}$$

$$x-2+\frac{4x+1}{x(x+2)}$$

vertical at $x=0$, $x=-2$

oblique at $x-2=y$



21. The illumination provided by a car's headlight varies inversely as the square of the distance from the headlight. A car's headlight produces an illumination of 3.75 foot-candles at a distance of 40 feet. What is the illumination when the distance is 50 feet? (6 points)

$$y=\frac{k}{d^2}$$

$$3.75=\frac{k}{40^2}$$

$$k=6000$$

$$y=\frac{6000}{d^2}$$

$$y=\frac{6000}{50^2}=2.4$$ ft candles