

MAT 142 Homework #1 Key

$$1. a. \frac{\frac{x}{3} - 1}{x - 3} \cdot \frac{(3)}{(3)} = \frac{\cancel{x-3}}{3(\cancel{x-3})} = \frac{1}{3}$$

$$b. \frac{\frac{1}{x+1}}{\frac{1}{x^2-2x-3} + \frac{1}{x-3}} \cdot \frac{(x+1)(x-3)}{(x+1)(x-3)} = \frac{x-3}{1+x+1} = \frac{x-3}{x+2}$$

$$c. \frac{1 + \frac{1}{x}}{3 - \frac{1}{x}} \cdot \frac{(x)}{(x)} = \frac{x+1}{3x-1}$$

$$d. \frac{x - \frac{x}{x+3}}{x+2} \cdot \frac{(x+3)}{(x+3)} = \frac{x^2+3x-x}{x^2+5x+6}$$

$$e. \frac{\frac{x+h}{x+h+1} - \frac{x}{x+1}}{h} = \frac{1}{h} \left[\frac{x+h}{x+h+1} - \frac{x}{x+1} \right] = \frac{x^2+2x}{x^2+5x+6} = \frac{x(x+2)}{(x+2)(x+3)} = \frac{x}{x+3}$$

$$= \frac{1}{h} \left[\frac{(x+h)(x+1) - x(x+h+1)}{(x+h+1)(x+1)} \right] = \frac{1}{h} \left[\frac{x^2+x+hx+h - x^2-xh-x}{(x+h+1)(x+1)} \right] =$$

$$\frac{1}{h} \left[\frac{h}{(x+h+1)(x+1)} \right] = \frac{1}{(x+h+1)(x+1)}$$

$$f. \frac{\frac{x}{x-2} + 1}{\frac{3}{x^2-4} + 1} = \frac{(x+2)(x-2)}{(x+2)(x-2)} = \frac{x(x+2) + x^2 - 4}{3 + x^2 - 4} = \frac{x^2 + 2x + x^2 - 4}{x^2 - 1} =$$

$$\frac{2x^2 + 2x - 4}{x^2 - 1} = \frac{2(x^2 + x - 2)}{x^2 - 1} = \frac{2(x+2)(x-1)}{(x-1)(x+1)} = \frac{2(x+2)}{x+1}$$

$$2a. \frac{7x+4}{-x-4} = \frac{x+16}{-x-4}$$

$$\frac{6x}{6} = \frac{12}{6}$$

$$\boxed{x=2}$$

$$b. 45 - [4 - 2y - 4(y+7)] = -4(1+3y) - [4 - 3(y+2) - 2(2y+5)]$$

$$45 - [4 - 2y - 4y - 28] = -4 - 12y - [4 - 3y - 6 - 4y - 10]$$

$$45 - [-6y - 24] = -4 - 12y - [-7y - 12]$$

$$45 + 6y + 24 = -4 - 12y + 7y + 12$$

$$6y + 69 = -5y + 8$$

$$+5y \quad -69 \quad +5y \quad -69$$

c. $\frac{x-2}{2x} + 1 = \frac{x+1}{x} \quad (2x) \Rightarrow x-2+2x = 2(x+1)$
 $3x-2 = 2x+2$
 $\frac{-2x+2 \quad -2x+2}{x = 4}$

d. $\frac{4}{x+5} + \frac{2}{x-5} = \frac{32}{x^2-25} \quad (x^2-25)$

$4(x-5) + 2(x+5) = 32 \Rightarrow 4x-20+2x+10 = 32 \Rightarrow 6x-10 = 32$
 $\frac{6x-10}{+10 \quad +10} = \frac{32}{+10 \quad +10} \Rightarrow 6x = 42 \Rightarrow x = 7$

e. $6x^2 + 11x - 10 = 0$

$(2x+5)(3x-2) = 0$
 $2x+5=0 \Rightarrow x = -\frac{5}{2}$
 $3x-2=0 \Rightarrow x = \frac{2}{3}$

f. $(5x-1)^2 = 7$

$5x-1 = \pm\sqrt{7}$
 $5x = 1 \pm \sqrt{7}$

$x = \frac{1 \pm \sqrt{7}}{5}$

g. $x^2 + 3x - 1 = 0$

$x = \frac{-3 \pm \sqrt{9+4}}{2} = \frac{-3 \pm \sqrt{13}}{2}$

h. $(2x-5)(x+1) = 2$

$2x^2 + 2x - 5x - 5 = 2$
 $2x^2 - 3x - 7 = 0$

$x = \frac{3 \pm \sqrt{9+4(2)7}}{4} = \frac{3 \pm \sqrt{65}}{4}$

i. $\frac{3}{x-3} + \frac{5}{x-4} = \frac{x^2-20}{x^2-7x+12} \quad (x-3)(x-4)$

$3(x-4) + 5(x-3) = x^2-20$
 $3x-12+5x-15 = x^2-20$

$8x-27 = x^2-20$
 $\frac{-8x+27 \quad -8x+27}{0 = x^2-8x+7}$
 $(x-1)(x-7) = 0$
 $x = 1, 7$

j. $\sqrt{2x+15} - 6 = x$

$(\sqrt{2x+15})^2 = (x+6)^2$

$2x+15 = x^2+12x+36$
 $\frac{-2x-15 \quad -2x-15}{0 = x^2+10x+21}$

k. $(\sqrt{1+4\sqrt{x}})^2 = (1+\sqrt{x})^2$

$1+4\sqrt{x} = 1+2\sqrt{x}+x$
 $(2\sqrt{x} = x)^2 \Rightarrow 4x = x^2$

$x^2-4x = 0 \Rightarrow x(x-4) = 0 \Rightarrow x = 0, x = 4$

$\sqrt{-6+15} = 3$
 $3-6 = -3\sqrt{\quad}$

$\sqrt{-7+15} = \sqrt{8}$
 $\sqrt{8}-6 = \sqrt{\quad}-7$

~~$(x+3)(x+7) = 0 \quad x = -3, x = -7$~~

$$2l. (x+3)^2 + 7(x+3) - 18 = 0$$

$$u = x+3$$

$$u^2 + 7u - 18 = 0$$

$$(u+9)(u-2) = 0$$

$$u = -9, u = 2$$

$$\begin{array}{r} x+3 = -9 \\ -3 \quad -3 \end{array} \quad \begin{array}{r} x+3 = 2 \\ \end{array}$$

$$x = -12$$

$$x = -1$$

$$\boxed{x = -12, -1}$$

$$n. |x-2| = 7$$

$$\begin{array}{r} x-2 = 7 \\ +2 \quad +2 \end{array}$$

$$\begin{array}{r} x-2 = -7 \\ +2 \quad +2 \end{array}$$

$$\boxed{x = 9}$$

$$\boxed{x = -5}$$

$$p. |x+1| + 6 = 2$$

$$|x+1| = -4$$

no solution

abs value cannot be negative

$$r. \frac{3}{x+3} = \frac{5}{2(x+3)} + \frac{1}{x-2} \quad 2(x+3)(x-2)$$

$$2 \cdot 3(x-2) = 5(x-2) + 2(x+3)$$

$$6x - 12 = 5x - 10 + 2x + 6$$

$$6x - 12 = 7x - 4$$

$$\begin{array}{r} -7x + 12 \\ + 12 \end{array}$$

$$-x = 8$$

$$\boxed{x = -8}$$

$$t. \frac{2x^2 - 5}{+5} = \frac{-55}{+5}$$

$$\frac{2x^2}{2} = \frac{-50}{2}$$

$$x^2 = -25$$

$$\boxed{x = \pm 5i}$$

$$m. x^{-2} - x^{-1} - 20 = 0$$

$$u = x^{-1} = \frac{1}{x}$$

$$u^2 - u - 20 = 0$$

$$(u-5)(u+4) = 0$$

$$u = 5, u = -4$$

$$\frac{1}{x} = 5 \Rightarrow \boxed{x = \frac{1}{5}}$$

$$\frac{1}{x} = -4 \Rightarrow \boxed{x = -\frac{1}{4}}$$

$$o. |3x-2| + 4 = 4$$

$$|3x-2| = 0$$

$$3x-2 = 0$$

$$\boxed{x = \frac{2}{3}}$$

$$q. \left(\frac{x+3}{6} = \frac{3}{8} + \frac{x-5}{4} \right) 24$$

$$4(x+3) = 9 + 6(x-5)$$

$$4x + 12 = 9 + 6x - 30$$

$$4x + 12 = 6x - 21$$

$$\begin{array}{r} -6x - 12 \\ -6x - 12 \end{array}$$

$$\frac{-2x}{-2} = \frac{-33}{-2} \Rightarrow x = \frac{33}{2}$$

$$x = \frac{33}{2}$$

$$s. x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0$$

$$\boxed{x = 5, -2}$$

$$u. (x-1)^2 = -9$$

$$x-1 = \pm 3i$$

$$\boxed{x = 1 \pm 3i}$$

$$v. (2x+3)(x+4) = 1$$

$$2x^2 + 8x + 3x + 12 = 1$$

$$2x^2 + 11x + 11 = 0$$

$$x = \frac{-11 \pm \sqrt{121 - 4(2)(11)}}{4} = \frac{-11 \pm \sqrt{33}}{4}$$

(3)

$$w. \frac{1}{x} + \frac{1}{x+3} = \frac{1}{4} (4x(x+3))$$

$$4(x+3) + 4x = x(x+3)$$

$$4x + 12 + 4x = x^2 + 3x$$

$$\begin{array}{r} 8x + 12 = x^2 + 3x \\ -8x - 12 \quad -8x - 12 \\ \hline \end{array}$$

$$0 = x^2 - 5x - 12$$

$$x = \frac{5 \pm \sqrt{25 + 48}}{2}$$

$$= \boxed{\frac{5 \pm \sqrt{73}}{2}}$$

$$z. 2x^4 = 16x$$

$$2x^4 - 16x = 0$$

$$2x(x^3 - 8) = 0$$

$$2x(x-2)(x^2 + 2x + 4) = 0$$

$$\boxed{x=0, x=2} \quad x = \frac{-2 \pm \sqrt{4-16}}{2}$$

$$= \frac{-2 \pm 2\sqrt{2}i}{2}$$

$$bb. x^{2/5} + x^{1/5} - 6 = 0$$

$$u = x^{1/5}$$

$$u^2 + u - 6 = 0$$

$$(u+3)(u-2) = 0$$

$$u = -3, u = 2$$

$$x^{1/5} = -3 \Rightarrow x = -243$$

$$x^{1/5} = 2 \Rightarrow x = 32$$

$$dd. \frac{7|3x| + 2}{-2 - 2} = 16$$

$$\frac{7|3x|}{7} = \frac{14}{7}$$

$$x. \sqrt{3x+18} = x$$

$$3x+18 = x^2$$

$$x^2 - 3x - 18 = 0$$

$$(x-6)(x+3) = 0$$

$$\boxed{x=6}, x=-3$$

$$\sqrt{18+18} = \sqrt{36} = 6 \quad (4)$$

$$\sqrt{-9+18} = \sqrt{9} = 3 \neq -3$$

$$y. \sqrt{3\sqrt{x+1}} = \sqrt{3x-5}$$

$$3\sqrt{x+1} = 3x-5$$

$$9(x+1) = 9x^2 - 30x + \frac{25}{9}$$

$$0 = 9x^2 - 39x + 16$$

$$x = \frac{+39 \pm \sqrt{39^2 - 4(16)9}}{18}$$

$$= \frac{39 \pm \sqrt{945}}{18} = \frac{39 \pm 3\sqrt{105}}{18} = \boxed{\frac{13 \pm \sqrt{105}}{6}}$$

$$aa. 2x - 3\sqrt{x} + 1 = 0$$

$$u = \sqrt{x}$$

$$2u^2 - 3u + 1 = 0 \quad (2u-1)(u-1) = 0$$

$$u = 1/2, u = 1$$

$$x = 1/4, x = 1$$

$$cc. \frac{2|3x-2|}{2} = \frac{14}{2}$$

$$|3x-2| = 7$$

$$\begin{array}{r} 3x-2 = 7 \\ +2 \quad +2 \end{array}$$

$$3x = 9$$

$$\boxed{x=3}$$

$$\begin{array}{r} 3x-2 = -7 \\ +2 \quad +2 \end{array}$$

$$3x = -5$$

$$\boxed{x = -\frac{5}{3}}$$

$$|3x| = 2$$

$$3x = 2$$

$$\boxed{x = 2/3}$$

$$3x = -2$$

$$\boxed{x = -2/3}$$