MAT 212, Quiz #10, Spring 2017 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round. Be sure to answer all parts of each question.

1. Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 2 & 3 \\ 4 & -2 \end{bmatrix}$.

$(2-\lambda)(-2-\lambda)-12=0$

$\lambda^2-4-12=0 \Rightarrow \lambda^2-16=0$

$\lambda=\pm 4$

$\lambda_1=4$

$\begin{bmatrix} -2 & 3 \\ 4 & -6 \end{bmatrix}$ $\vec{v}_1=\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$-2x_1+3x_2=0$

$x_1=\frac{3x_2}{2}$

$x_2=x_2$

$\lambda_2=-4$

$2\div\begin{bmatrix} 6 & 3 \\ 4 & 2 \end{bmatrix}$ $\vec{v}_2=\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$2x_1+x_2=0$

$x_1=-\frac{1}{2}x_2$

$x_2=x_2$

2. Find all the singular points of the equation $(1+x^3)y''+(x+1)^3y'+x^4y=0$. Characterize each as regular or irregular.

$$y''+\frac{(x+1)^3}{1+x^3}y'+\frac{x^4}{1+x^3}y=0$$

$$y''+\frac{(x+1)^2}{x^2+x+1}y'+\frac{x^4}{1+x^3}y=0$$

defined so limit exist after cancelling

$x=-1$ is singular

$$\lim_{x\to -1}\frac{x^4(x+1)^2}{1+x^3}=\lim_{x\to -1}\frac{x(x+1)}{x^2+x+1}=\text{exists}$$

$x=-1$ is regular

3. Solve $y''+x^2y+2xy=0$ by series methods.

3. $y=\sum_{n=0}^{\infty} a_n x^n \qquad y'=\sum_{n=1}^{\infty} n a_n x^{n-1} \qquad y''=\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} + x^2\sum_{n=1}^{\infty} n a_n x^{n-1} + 2x\sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} + \sum_{n=1}^{\infty} n a_n x^{n+1} + \sum_{n=0}^{\infty} a_n x^{n+1} = 0$$

$$\sum_{n=-1}^{\infty} (n+3)(n+2)a_{n+3} x^{n+1} + \sum_{n=1}^{\infty} n a_n x^{n+1} + \sum_{n=0}^{\infty} a_n x^{n+1} = 0$$

$$\underbrace{(2)(1)a_2(1)}_{n=-1} + \underbrace{(3)(2)a_3 x}_{n=0} + \sum_{n=1}^{\infty}(n+3)(n+2)a_{n+3}x^{n+1} + \sum_{n=1}^{\infty} n a_n x^{n+1} + \underbrace{a_0(x)}_{n=0} + \sum_{n=1}^{\infty} a_n x^{n+1} = 0$$

$2a_2 = 0 \qquad 6a_3 + a_0 = 0$

$a_3 = -\frac{1}{6}a_0$

$$\sum_{n=1}^{\infty}\left[(n+3)(n+2)a_{n+3} + \underbrace{n a_n + a_n}_{a_n(n+1)}\right]x^{n+1} = 0$$

$$a_{n+3} = \frac{-(n+1)a_n}{(n+3)(n+2)}$$

$n=0 \qquad a_3 = \frac{-1a_0}{(3)(2)}$

$n=1 \qquad a_4 = \frac{-(2)a_1}{\cancel{2}4(3)} = -\frac{a_1}{6}$

$n=2 \qquad a_5 = \frac{-3(a_2)}{5(4)} = 0$

$n=3 \qquad a_6 = \frac{-4(a_3)}{6(5)} = \frac{1}{90}a_0$

$n=4 \qquad a_7 = \frac{-5(a_4)}{7(6)} = \frac{5}{252}a_1$

$n=5 \qquad a_8 = 0$

$n=6 \qquad a_9 = \frac{-7(a_6)}{9(8)} = -\frac{7}{6480}a_0$

$n=7 \qquad a_{10} = \frac{-8(a_7)}{10(9)} =$

$n=8$

$a_{11} = 0$

a_0	a_1	$a_2=0$
$a_3=-\frac{1}{6}a_0$	$a_4=-\frac{1}{6}a_1$	$a_5=0$
$a_6=\frac{1}{90}a_0$	$a_7=\frac{5}{252}a_1$	$a_8=0$
$a_9=\frac{-7}{6480}a_0$	$a_{10}=\frac{-1}{567}a_1$	$a_{11}=0$
$\vdots$	$\vdots$	$\vdots$

$$y(x) = a_0\left[1-\frac{1}{6}x^3+\frac{1}{90}x^6-\frac{7}{6480}x^9+\cdots\right] + a_1\left[x-\frac{1}{6}x^4+\frac{5}{252}x^7-\frac{1}{567}x^{10}+\cdots\right]$$