MAT 212, Quiz #14, Spring 2017 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round. Be sure to answer all parts of each question.

1. Write a system of equations that models a 2-tank system where Tank A has 1000L of brine solution containing 100kg of salt to start, and Tank B has 2000L with no salt to start. Pure water flows into Tank A at 4L/s and the mixture flows into Tank B at the same rate.

$\frac{dA}{dt} = -\frac{A}{1000} \cdot 4 = -\frac{A}{250}$

$A(0) = 100$
$B(0) = 0$

$\frac{dB}{dt} = \frac{A}{250} = \frac{A}{250}$

$$\begin{bmatrix} A \\ B \end{bmatrix}' = \begin{bmatrix} -\frac{1}{250} & 0 \\ \frac{1}{250} & 0 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

2. Estimate the solution to $y' = 2xy, y(0) = 2$ at the point $y(1)$ in 3 steps using improved Eulers:

$$k_1 = f(t_n, y_n)$$
$$u_{n+1} = y_n + \Delta t\, k_1$$
$$k_2 = f(t_{n+1}, u_{n+1})$$
$$y_{n+1} = y_n + \frac{1}{2}\Delta t(k_1 + k_2)$$

$\Delta x = \frac{1}{3}$

Compare your result to the true solution $y = 2e^{x^2}$.

$x_0 = 0$ $y_0 = 2$ $k_{10} = 2(0)(2) = 0$ $u_1 = 2 + \frac{1}{3}(0) = 2$
$k_{20} = 2(0)(2) = 0$ $y_1 = 2 + \frac{1}{2}\cdot\frac{1}{3}(0+0) = 2$

$x_1 = \frac{1}{3}$ $y_1 = 2$ $k_{11} = 2(\frac{1}{3})(2) = \frac{4}{3}$ $u_2 = 2 + \frac{1}{3}(\frac{4}{3}) = \frac{22}{9}$
$k_{21} = 2(\frac{1}{3})(\frac{22}{9}) = \frac{44}{27}$ $y_2 = 2 + \frac{1}{2}(\frac{1}{3})(\frac{4}{3} + \frac{44}{27}) = \frac{202}{81}$

$x_2 = \frac{2}{3}$ $y_2 = \frac{202}{81}$ $k_{12} = (2)(\frac{2}{3})(\frac{202}{81}) = \frac{808}{243}$ $u_3 = \frac{202}{81} + \frac{1}{3}(\frac{808}{243}) = \frac{4626}{729}$
$k_{22} = 2(\frac{2}{3})(\frac{4626}{729}) = \frac{10504}{2187}$ $y_3 = \frac{202}{81} + \frac{1}{2}\cdot\frac{1}{3}(\frac{808}{243} + \frac{10504}{2187}) =$

$x_3 = 1$ $y_3 = \frac{25250}{6561} \approx 3.848$

$y(1) = 5.43656\ldots \approx 2e^1$