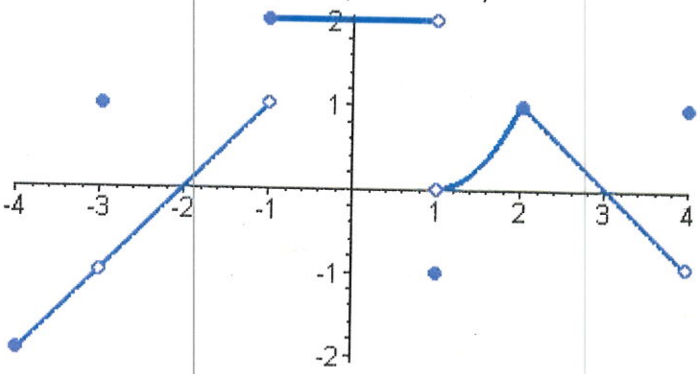
MAT 230, Exam #1, Part I, Spring 2017 Name KEY

Instructions: Show all work, and provide exact answers. For full credit will be given to the steps shown than for the final answer. Be sure to provide thorough explanations. On this portion of the exam, **no calculator is permitted**.

1. Consider the graph of the piecewise function $f(x)$ below and use the graph to answer the questions that follow. (a-l: 2 points each)



a. $\lim_{x\to -1^+} f(x)$ $= 2$

b. $\lim_{x\to -1^-} f(x)$ 1

c. $\lim_{x\to -1} f(x)$ DNE

d. $\lim_{x\to 2^+} f(x)$ 1

e. $\lim_{x\to 2^-} f(x)$ 1

f. $\lim_{x\to 2} f(x)$ 1

g. $\lim_{x\to -3^+} f(x)$ -1

h. $\lim_{x\to -3^-} f(x)$ -1

i. $\lim_{x\to -3} f(x)$ -1

j. $f(-1)$ 2

k. $f(2)$ 1

l. $f(-3)$ 1

m. What can you conclude about the continuity of $f(x)$ at $x = -1, x = -3$, and $x = 2$, based on the calculations above? (3 points)

the limit at -1 does not exist so it is not continuous there
the limit and the function are not the same value at 3, so
not continuous there; the limit exists & function is the same
at 2, so it is continuous there

2. Find the limit. (5 points each)

a. $\lim_{x \to 3} (x^2 - 4x + 7)$

$= 9 - 12 + 7 = 4$

b. $\lim_{x \to 1} \frac{x^3-1}{x-1}$ $= \lim_{x \to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}} = \lim_{x \to 1} (x^2+x+1) = 3$

3. Use the limit definition of the derivative to find $f'(x)$ for each $f(x)$. (10 points each)

a. $f(x) = x^2 + 3x - 2$

$\lim_{h \to 0} \frac{(x+h)^2 + 3(x+h) - 2 - (x^2+3x-2)}{h} = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{3x} + 3h \cancel{-2} \cancel{-x^2} \cancel{-3x} + \cancel{2}}{h}$

$= \lim_{h \to 0} \frac{2xh + h^2 + 3h}{2} = \lim_{h \to 0} \frac{\cancel{h}(2x + h + 3)}{\cancel{h}} = \lim_{h \to 0} 2x + h + 3 =$

$2x + 3$

b. $f(x) = \frac{2}{x}$

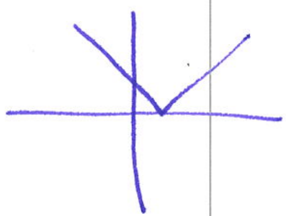
$\lim_{h \to 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h} = \lim_{h \to 0} \frac{1}{h}\left(\frac{2x}{x(x+h)} - \frac{2(x+h)}{x(x+h)}\right) =$

$\lim_{h \to 0} \frac{\cancel{2x} - \cancel{2x} - 2h}{hx(x+h)} = \lim_{h \to 0} \frac{-2\cancel{h}}{\cancel{h}x(x+h)} = \lim_{h \to 0} \frac{-2}{x(x+h)}$

$= \frac{-2}{x^2}$

4. Sketch the graph of a function that is not differentiable everywhere, and explain why it is not differentiable at that point. (5 points)

answers will vary - may have a cusp or may be discontinuous



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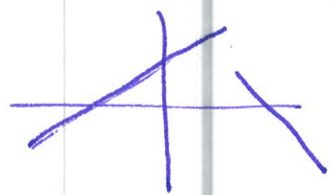


etc.

5. Use L'Hôpital's Rule to find $\lim_{x \to 5} \left(\frac{x^2-25}{2x-10}\right)$. (6 points)

$\frac{0}{0}$

$$\lim_{x \to 5} \left(\frac{2x}{2}\right) = \lim_{x \to 5} x = 5$$

MAT 230, Exam #1, Part II, Spring 2017

Name KEY

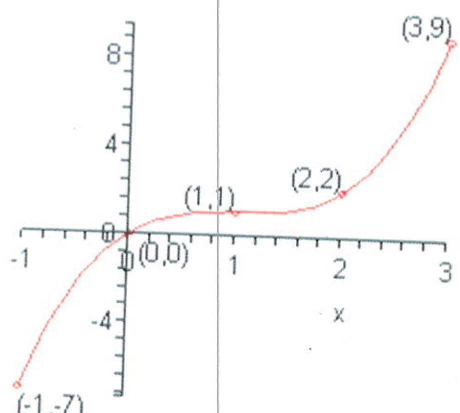
Instructions: Show all work, and provide exact answers. For full credit will be given to the steps shown than for the final answer. Be sure to provide thorough explanations. You may use a calculator on this portion of the exam. If you use your calculator, describe the steps you used, or sketch the graph obtained from your calculator to show work.

1. Build a table of values to find the value of the limit of $\lim_{x\to 1}\left(\frac{x-\sqrt[4]{x}}{x-1}\right)$. (6 points)

x	y
0	0
.9	.74
.99	.749
.999	.7499
.9999	.74999
2	.81
1.1	.75886
1.01	.75093
1.001	.75009
1.0001	.75001

the limit is 3/4

2. Find the average rate of change between $(-1,-7)$ and $(3,9)$. (6 points)



$$\frac{9-(-7)}{3-(-1)} = \frac{16}{4} = 4$$

3. Differentiate each function. (6 points each)

a. $y = \frac{1}{2}x^{4/5}$

$y' = \frac{1}{2}\left(\frac{4}{5}\right)x^{-1/5} = \frac{2}{5}x^{-1/5}$

b. $y = 5x^2 - 3x + 8$

$y' = 10x - 3$

c. $y = \ln(x^2)$ [Hint: simplify first.] $= 2\ln x$

$y = \frac{2}{x}$

d. $y = \frac{7}{x^3}$ $= 7x^{-3}$

$y' = -21x^{-4} = \frac{-21}{x^4}$

e. $y = 7e^{-x}$

$y' = -7e^{-x}$

4. Use the product, quotient or chain rule to differentiate. You do not need to simplify. (8 points each)

a. $f(t) = (3t^5 - t^2)\left(t - \frac{5}{t}\right)$

$f' \quad (15t^4 - 2t)(t - \frac{5}{t}) + (3t^5 - t^2)(1 + \frac{5}{t^2})$

b. $f(x) = \frac{x-1}{x+x^{-2}}$

$$y' = \frac{(1)(x+x^{-2}) - (1-2x^{-3})(x-1)}{(x+x^{-2})^2}$$

c. $f(x) = (3 + x^3)^5 + \frac{7}{8}e^{-x^2-5x}$

$5(3+x^3)^4(3x^2) + \frac{7}{8}e^{-x^2-5x}(-2x-5)$

d. $f(x) = \frac{x^2}{(1+x)^5}$ $\quad = x^2(1+x)^{-5}$

$$\frac{2x(1+x)^5 - x^2(5)(1+x)^4}{(1+x)^{10}} \quad \text{or} \quad 2x(1+x)^{-5} - 5x^2(1+x)^{-6}$$

5. Find the equation of the tangent line to the graph of $y = x + \frac{2}{x^3}$ at $x = 1$. (8 points)

$y' = 1 - \frac{6}{x^4}$ $\quad y'(1) = 1 - 6 = -5$ $\quad x + 2x^{-3}$ $\quad y(1) = 1 + 2 = 3$

$y - 3 = -5(x - 1) \Rightarrow y - 3 = -5x + 5$

$\qquad +3 \qquad +3$

$y = -5x + 8$

6. Find $\frac{d^3y}{dx^3}$ for $y = 2e^{3x} - \ln(x) + 9x^4 - 8\sqrt{x^3}$. (15 points)

$8x^{3/2}$

$$y' = 6e^{3x} - \frac{1}{x} + 36x^3 - 12x^{1/2}$$

(x^{-1})

$$y'' = 18e^{3x} + \frac{1}{x^2} + 108x^2 - 6x^{-1/2}$$

(x^{-2}) $\frac{6}{\sqrt{x}}$

$$y''' = 54e^{3x} - \frac{2}{x^3} + 216x + 3x^{-3/2}$$

$$= 54e^{3x} - \frac{2}{x^3} + 216x + \frac{3}{\sqrt{x^3}}$$