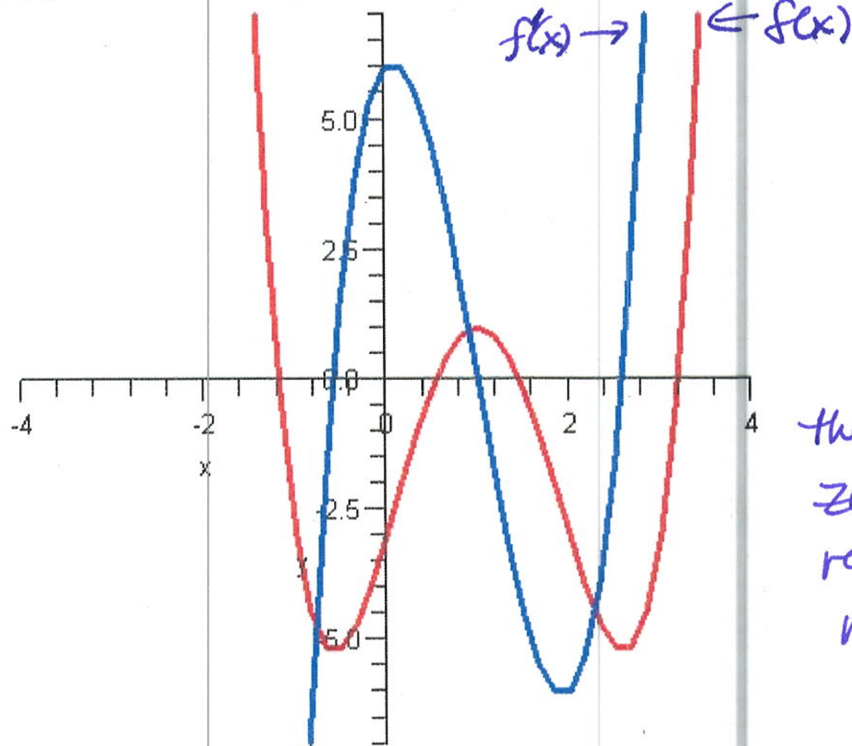
MAT 230, Exam #2, Part I, Spring 2017 Name KEY

Instructions: Show all work, and provide exact answers. For full credit will be given to the steps shown than for the final answer. Be sure to provide thorough explanations. On this portion of the exam, **no calculator is permitted**.

1. The graph below shows two functions. One function $f'(x)$ is the derivative of the other function $f(x)$. Determine which graph is which, and label both. (6 points)



the blue graph is zero whenever the red graph has a max or min.

2. Consider the function $f(x) = x^4 - 4x^3 + 10$. Final all the extrema and inflection points. Classify all extrema. Use that information to sketch the curve. (15 points)

$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$ $x=0, x=3$

$f''(x) = 12x^2 - 24x = 12x(x-2)$ $x=0, x=2$

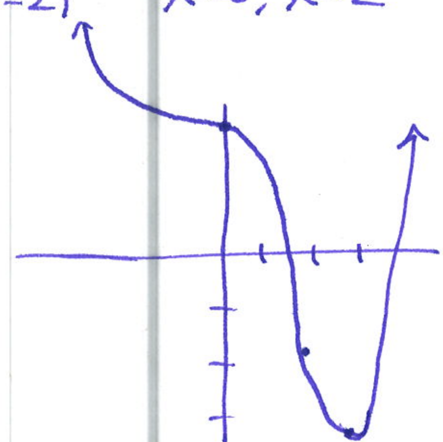
f': $-$ | 0 | $-$ | 3 | $+$

f'': $+$ | 0 | $-$ | 2 | $+$

$f(0)=10$

$f(3)=-17$

$f(2)=-6$



3. Final all asymptotes, intercepts and critical points, and use that information to graph $f(x) = \frac{x-2}{x+1}$. (10 points)

vertical at $x=-1$

x-intercept at $x=2$

horizontal at $y=1$

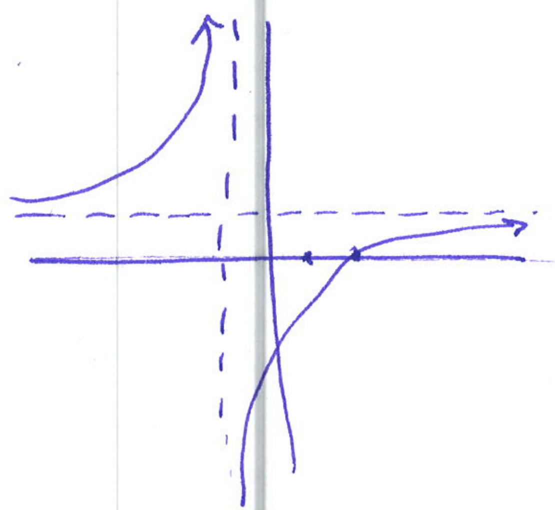
$$y' = \frac{1(x+1)-(x-2)(1)}{(x+1)^2}$$

$$= \frac{x+1-x+2}{(x+1)^2} = \frac{3}{(x+1)^2}$$

always positive

always increasing

(except $x=-1$)



4. Find $\frac{dy}{dx}$ for $3x^2y^4 = 12$. (8 points)

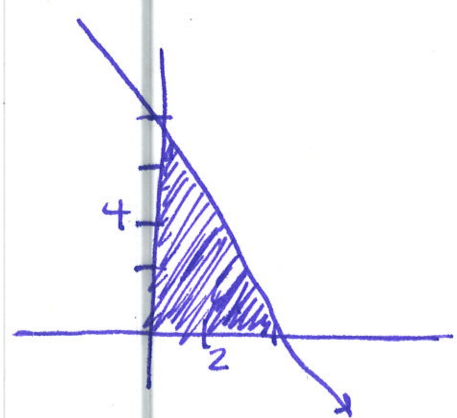
$$6xy^4 + 12x^2y^3y' = 0$$

$$12x^2y^3y' = -6xy^4$$

$$y' = \frac{-6xy^4}{12x^2y^3} = \frac{-y}{2x}$$

5. Geometrically find the area under the curve $y = 4 - 2x$ on $[0,2]$. [Hint: Sketch the graph. What shape is this?] (8 points)

$$A = \tfrac{1}{2}bh = \tfrac{1}{2}(2)(4) = 4$$



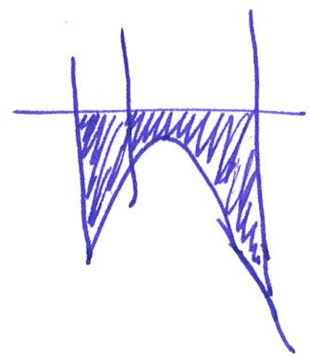
6. Evaluate $\int_{-2}^{3} -x^2 + 4x - 5\,dx$. (8 points)

$$-\frac{1}{3}x^3 + 2x^2 - 5x \Big|_{-2}^{3} =$$

$$-\frac{1}{3}(27) + 2(9) - 5(3) - \left[-\frac{1}{3}(-8) + 2(4) - 5(-2)\right]$$

$$= -9 + 18 - 15 - \left[\frac{8}{3} + 8 + 10\right] =$$

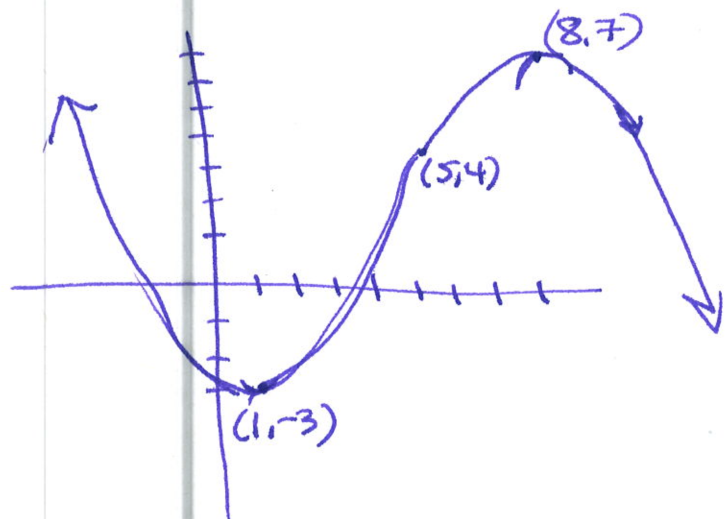
$$-6 - \left[\frac{62}{3}\right] = -\frac{80}{3}$$



MAT 230, Exam #2, Part II, Spring 2017 Name KEY

Instructions: Show all work, and provide exact answers. For full credit will be given to the steps shown than for the final answer. Be sure to provide thorough explanations. You may use a calculator on this portion of the exam. If you use your calculator, describe the steps you used, or sketch the graph obtained from your calculator to show work.

1. Sketch the graph of a function that is concave up at $(1,-3)$, concave down at $(8,7)$, and has an inflection point at $(5,4)$. (6 points)



2. Find the absolute extrema of the function on the given interval. (5 points each)

a. $f(x) = 24, [4,13]$

function is constant. every point on the interval is the same and both the max & min

$f'(x) = 0$

b. $f(x) = 1 - x^{2/3}, [-8,8]$

$f' = -\frac{2}{3}x^{-1/3} = 0$?

never zero

$\frac{-2}{3\sqrt[3]{x}}$ undefined at $x=0$ critical point

test $-8, 8, 0$

$f(-8) = -3$ } abso min

$f(8) = -3$

$f(0) = 1 \leftarrow$ abso max

c. $f(x) = x + \frac{1}{x}$, [1,20] $f'(x) = 1 - \frac{1}{x^2} = 0$ $x = \pm 1$

-1 not in interval

Check 1, 20

$f(1) = 2 \leftarrow$ abs min

$f(20) = 20.05 \leftarrow$ abs max

3. If the revenue function for a product is $R(x) = 50x - \frac{1}{2}x^2$, and the cost function is $C(x) = 4x + 10$, maximize the profit. (9 points)

profit = Rev - cost = $50x - \frac{1}{2}x^2 - (4x + 10) =$

$50x - \frac{1}{2}x^2 - 4x - 10 =$

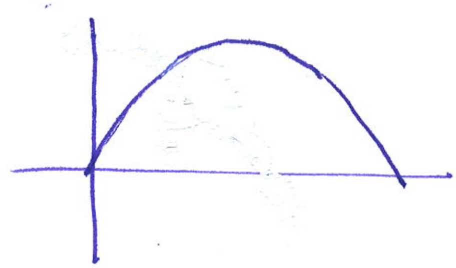
$-\frac{1}{2}x^2 + 46x - 10$

Profit

$P' = -x + 46$

$x = 46$

max profit $P(46) = 1048$



4. Suppose the price p and sales x of a mechanical pencil are related by the equation $5p + 4x + 2px = 60$. Find the rate of change of sales when $x = 3, p = 5, \frac{dp}{dt} = 1.5$. (8 points)

dx/dt

$$5\frac{dp}{dt} + 4\frac{dx}{dt} + 2\frac{dp}{dt}x + 2p\frac{dx}{dt} = 0$$

$$5(1.5) + 4\left(\frac{dx}{dt}\right) + 2(1.5)(3) + 2(5)\left(\frac{dx}{dt}\right) = 0$$

$$7.5 + [4 + 10]\frac{dx}{dt} + 9 = 0$$

$$14\frac{dx}{dt} = -16.5$$

$$\frac{dx}{dt} = -\frac{33}{28}$$

5. Integrate.

a. $\int 4\sqrt[5]{x} + \frac{3}{4}e^{6x} - \frac{7}{x}dx$ (8 points)

$$\int 4x^{1/5} + \frac{3}{4}e^{6x} - \frac{7}{x}\,dx = 4\left(\frac{5}{6}\right)x^{6/5} + \frac{3}{4}\left(\frac{1}{6}\right)e^{6x} - 7\ln x + C$$

$$\frac{10}{3}x^{6/5} + \frac{1}{8}e^{6x} - 7\ln x + C$$

b. $\int xe^{x^2} - \frac{\ln x}{x} + (6x^2-1)(2x^3-x+11)^7 dx$ (12 points)

$\int xe^{x^2}dx$ $\quad u = x^2$, $du = 2x\,dx$, $\frac{1}{2}du = x\,dx$

$\int \frac{1}{2}e^u du = \frac{1}{2}e^u$

$\int -\frac{\ln x}{x}dx$ $\quad u = \ln x$, $du = \frac{1}{x}dx$

$-\int u\,du = -\frac{1}{2}u^2$

$\int (6x^2-1)(2x^3-x+11)^7 dx$

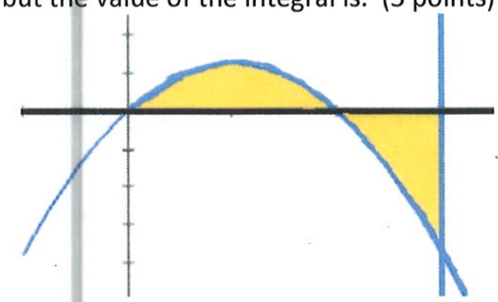
$u = (2x^3-x+11)$

$du = 6x^2-1\,dx$

$\int u^7 du = \frac{1}{8}u^8$

$$\frac{1}{2}e^{x^2} - \frac{1}{2}(\ln x)^2 - \frac{1}{8}(2x^3-x+11)^8 + C$$

6. The graph shown below illustrates the integral $\int_0^{3/2} x - x^2 dx$. The value of this integral is zero. Explain why the area bounded by the curve is not zero, but the value of the integral is. (5 points)



the area above the x-axis and the area below the x-axis are the same, but for the sign, so they cancel out. to calculate the area one needs to take the absolute value of each part before adding

7. Estimate the area under the curve $f(x) = x^2 + 1$ on $[1,3]$ with 4 rectangles, using the right-hand rule. (15 points)

$$\sum_{i=1}^{4} f(x_i)\Delta x_i$$

$$\frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$$

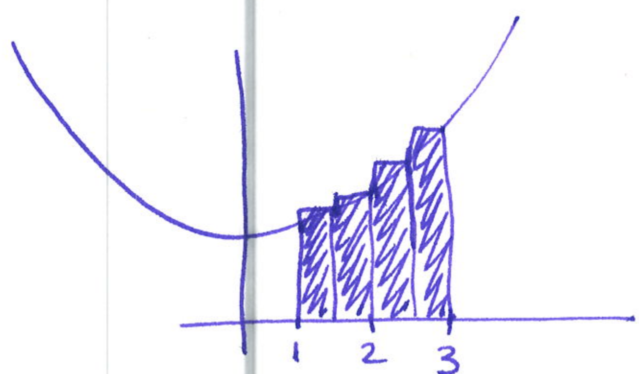
$$\underbrace{x_0 = 1, x_1 = \tfrac{3}{2}, x_2 = 2, x_3 = \tfrac{5}{2}, x_4 = 3}_{\text{use these for right-hand rule}}$$

$$\Delta x_i = \tfrac{1}{2}$$

$$\frac{1}{2}\left[\left(\tfrac{3}{2}^2+1\right) + \left(2^2+1\right) + \left(\tfrac{5}{2}^2+1\right) + \left(3^2+1\right)\right] =$$

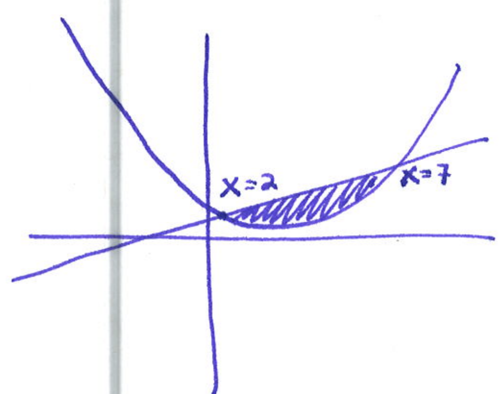
$$\frac{1}{2}\left[3.25 + 5 + 7.25 + 10\right] =$$

$$\frac{1}{2}\left[25.5\right] = 12.75$$



8. Set up an integral to find the area between the curves $f(x) = x^2 - 7x + 20$, $g(x) = 2x + 6$. You do not need to evaluate it. Sketch the graph. (8 points)

$$x^2 - 7x + 20 = 2x + 6$$
$$x^2 - 9x + 14 = 0$$
$$(x-7)(x-2) = 0$$
$$x = 7,\ x = 2$$



$$\int_2^7 (2x+6) - (x^2 - 7x + 20)\,dx$$

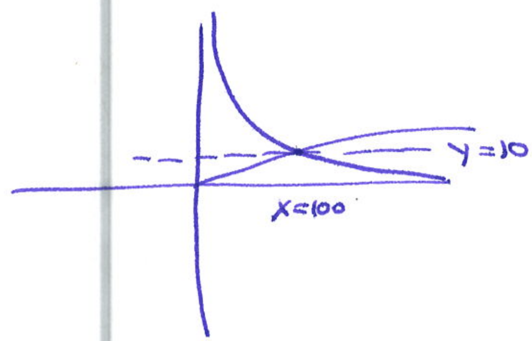
$$= \int_2^7 2x + 6 - x^2 + 7x - 20\,dx$$

$$= \int_2^7 9x - 14 - x^2\,dx$$

9. If the demand function is $D(x) = \frac{100}{\sqrt{x}}$, and the supply function is $S(x) = \sqrt{x}$, set up the integrals for consumer's and producer's surplus. You do not need to evaluate them. (10 points)

$$\sqrt{x}\left(\frac{100}{\sqrt{x}} = \sqrt{x}\right) \Rightarrow 100 = x$$

$$D(100) = 10$$



Consumer's surplus

$$\int_0^{100} \frac{100}{\sqrt{x}} - 10 \, dx$$

producer's surplus

$$\int_0^{100} 10 - \sqrt{x} \, dx$$

10. Verify that $y = -2e^x + xe^x$ is a solution to the differential equation $y'' - 2y' + y = 0$. (7 points)

$$y' = -2e^x + e^x + xe^x$$
$$= -e^x + xe^x$$
$$y'' = -e^x + e^x + xe^x$$
$$= xe^x$$

$$xe^x - 2(-e^x + xe^x) + (-2e^x + xe^x)$$
$$= xe^x + 2e^x - 2xe^x - 2e^x + xe^x$$
$$= 0 \checkmark$$

11. Find y if $\frac{dy}{dx} = \frac{2x}{y}$, and $y(0) = 11$. (7 points)

$$\int y \, dy = \int 2x \, dx$$
$$\frac{1}{2}y^2 = x^2 + C$$
$$y^2 = 2x^2 + C$$
$$11^2 = 0 + C$$
$$C = 121$$

$$y^2 = 2x^2 + 121$$
or $y = \pm\sqrt{2x^2 + 121}$