MAT 230 Written Homework #7 Key

1a. $\sum_{i=1}^{n} f(x_i)\Delta x_i$ $\quad \frac{7-1}{4} = \frac{6}{4} = \frac{3}{2} = \Delta x_i$ right hand rule

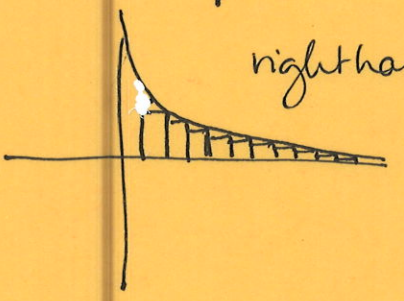
$f(x) = \frac{1}{x^2}$, $[1,7]$ $x_0 = 1$, $x_1 = 2.5$, $x_2 = 4$, $x_3 = 5.5$, $x_4 = 7$

$\sum_{i=1}^{4} f(x_i)\left(\frac{3}{2}\right) =$ $\quad f(x_1) = \frac{1}{2.5^2}$ $f(x_2) = \frac{1}{4^2}$, $f(x_3) = \frac{1}{5.5^2}$, $f(x_4) = \frac{1}{7^2}$

$\sum_{i=1}^{4} \frac{1}{x_i^2}\left(\frac{3}{2}\right) =$

$\frac{3}{2}\left[\frac{1}{2.5^2} + \frac{1}{16} + \frac{1}{5.5^2} + \frac{1}{7^2}\right] = 0.483866\ldots$

1b. $\Delta x_i = \frac{7-1}{10} = \frac{6}{10} = .6$



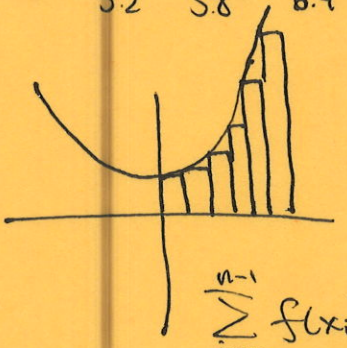
$x_0 = 1$, $x_1 = 1.6$, $x_2 = 2.2$, $x_3 = 2.8$, $x_4 = 3.4$,
$x_5 = 4.0$, $x_6 = 4.6$, $x_7 = 5.2$, $x_8 = 5.8$,
$x_9 = 6.4$, $x_{10} = 7.0$

$f(x_i) = \frac{1}{x_i^2}$

$\sum_{i=1}^{10} f(x_i)\left(\frac{3}{5}\right) = \frac{3}{5}\left[\frac{1}{1.6^2} + \frac{1}{2.2^2} + \frac{1}{2.8^2} + \frac{1}{3.4^2} + \frac{1}{16} + \frac{1}{4.6^2} + \frac{1}{5.2^2} + \frac{1}{5.8^2} + \frac{1}{6.4^2} + \frac{1}{7^2}\right] = 0.619549\ldots$

c. $f(x) = x^2 + 1$, $[0,5]$ $\quad n = 6$

$\Delta x_i = \frac{5-0}{6} = \frac{5}{6}$



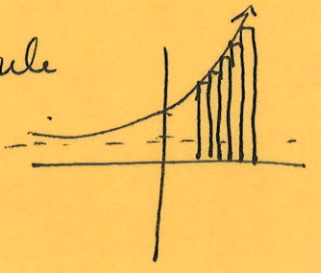
$x_0 = 0$, $x_1 = \frac{5}{6}$, $x_2 = \frac{10}{6}$, $x_3 = \frac{15}{6}$,
$x_4 = \frac{20}{6}$, $x_5 = \frac{25}{6}$, $x_6 = \frac{30}{6} = 5$

$\sum_{i=0}^{n-1} f(x_i)\Delta x_i$

$\sum_{i=0}^{5} f(x_i)\Delta x_i = \frac{5}{6}\left[(0^2+1) + \left(\left(\frac{5}{6}\right)^2+1\right) + \left(\left(\frac{10}{6}\right)^2+1\right) + \left(\left(\frac{15}{6}\right)^2+1\right) + \left(\left(\frac{20}{6}\right)^2+1\right) + \left(\left(\frac{25}{6}\right)^2+1\right)\right]$

$= 36.8287$

d. $f(x) = e^x + 1$, $[1,2]$, $n = 5$ $\quad$ left hand rule



$\Delta x_i = \frac{2-1}{5} = \frac{1}{5} = .2$ $\quad f(x_0) = e+1$, $f(x_1) = e^{1.2}+1$, $f(x_2) = e^{1.4}+1$
$f(x_3) = e^{1.6}+1$, $f(x_4) = e^{1.8}+1$

$\sum_{i=0}^{4} f(x_i)x_i = \frac{1}{5}\left[e+1 + e^{1.2}+1 + e^{1.4}+1 + e^{1.6}+1 + e^{1.8}+1\right] = 5.2192557\ldots$

#7 cont'd

1e. $f(x) = \ln(x^3 - 1)$, $[3, 8]$, $n = 3$ midpoint rule

$\Delta x_i = \frac{8-3}{3} = \frac{5}{3}$

$x_0 = 3$, $x_1 = \frac{14}{3}$, $x_2 = \frac{19}{3}$, $x_3 = 8$

$\frac{23}{6}$ $\quad$ 5.5 $\quad$ $\frac{43}{6}$

$$\frac{5}{3}\left[\ln\left(\left(\tfrac{23}{6}\right)^3 - 1\right) + \ln\left((5.5)^3 - 1\right) + \ln\left(\left(\tfrac{43}{6}\right)^3 - 1\right)\right] =$$

25.045