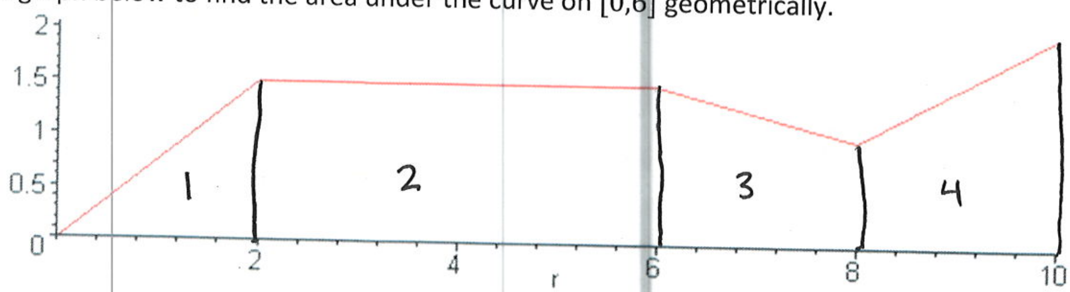
MAT 230, Written Homework #8, Spring 2017 Name ______________________________

**Instructions**: Show all work, and provide exact answers. Far for credit will be given to the steps shown than for the final answer. Be sure to provide thorough explanations. Some problems will require the use of technology (such as Excel, your calculator, or free widgets online). Complete your work on a separate page and attach to this cover sheet.

1. Evaluate the definite integrals and compare the results obtained from the approximations on the last homework.
   a. $f(x) = \frac{1}{x^2} \cdot [1,7]$
   b. $f(x) = x^2 + 1, [0,5]$
   c. $f(x) = e^x + 1, [1,2]$
   d. $f(x) = \ln(x^3 - 1), [3,8]$ (Do this integral numerically on your calculator or with a computer algebra system.)

2. Use the graph below to find the area under the curve on $[0,6]$ geometrically.




3. Sketch the set of curves and find the area bounded between them.
   a. $y = x, y = \sqrt[4]{x}$
   b. $y = x^2 + 1, y = x^2, x = 1, x = 3$
   c. $y = 3, y = x, x = 0$
   d. $y = x + 6, y = -2x, y = x^3$

4. Integrate. If an initial value is given, find the constant of integration.
   a. $\int \frac{1}{1+t} dt, f(0) = 1$
   b. $\int x e^{x^2} dx$
   c. $\int \frac{e^t}{3+e^t} dt$
   d. $\int \frac{4}{x \ln x} dx$
   e. $\int x\sqrt{x+1} dx, f(0) = 4$

MAT 230 Written Homework #8 Key 

1. a. $f(x) = \frac{1}{x^2}$ $[1,7]$ $\Rightarrow$ $\int_1^7 \frac{1}{x^2}dx = \int_1^7 x^{-2}dx = -\frac{1}{x}\Big|_1^7 = -\frac{1}{7}+\frac{1}{1} =$

$= .8571\ldots$

The second approx (1b) was closer w/ more rectangles. both underestimate

b. $f(x) = x^2+1$ $[0,5]$ $\Rightarrow$ $\int_0^5 x^2+1\,dx = \frac{x^3}{3}+x\Big|_0^5 = \frac{125}{3}+5-0 =$

$46.66\bar{6}$

compared to 36.8287 also underestimates

c. $f(x) = e^x+1$, $[1,2]$ $\Rightarrow$ $\int_1^2 e^x+1\,dx = e^x+x\Big|_1^2 = e^2+2-e-1$

$\approx 5.67$

Compared to 5.219... underestimates

d. $f(x) = \ln(x^3-1)$ $[3,8]$ $\Rightarrow$ $\int_3^8 \ln(x^3-1)\,dx = 24.97\ldots$

Compared to 25.045 overestimate but pretty close.

2. area 1: triangle $\frac{1}{2}bh = A$

$\frac{1}{2}(2)(1.5) = 1.5$

area 2: rectangle $A = lw = 4(1.5) = 6$

area 3: trapezoid $A = \frac{1}{2}(2)(1.5+1) = 2.5$

area : trapezoid $A = \frac{1}{2}(2)(1+2) = 3$

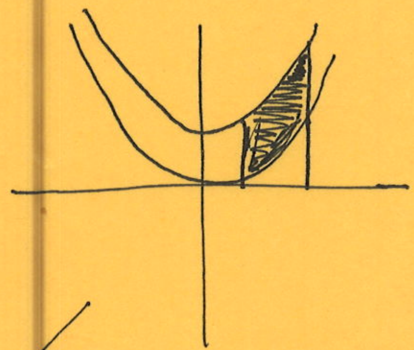
add areas $= 1.5+6+2.5+3 = 9+4 = 13$

3. a. $y = x$, $y = \sqrt[4]{x}$


$\int_0^1 \sqrt[4]{x} - x\,dx = \int_0^1 x^{1/4} - x\,dx =$

$\frac{4}{5}x^{5/4} - \frac{1}{2}x^2\Big|_0^1 = \frac{4}{5} - \frac{1}{2} = \frac{3}{10}$

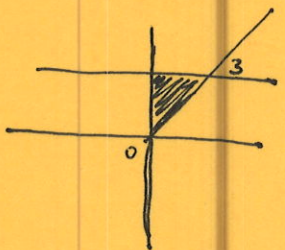
3b. $y = x^2+1,\ y = x^2,\ x=1,\ x=3$



$\int_1^3 (x^2+1) - x^2\,dx = \int_1^3 1\,dx = x\Big|_1^3 =$

$3-1=2$

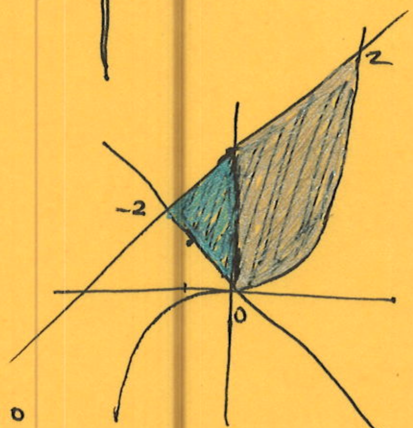
c. $y=3,\ y=x,\ x=0$




$\int_0^3 3-x\,dx = 3x-\frac{1}{2}x^2\Big|_0^3$

$= 9-\frac{9}{2} = \frac{9}{2}$

d. $y=x+6,\ y=-2x,\ y=x^3$




region 1: $\int_{-2}^{0} x+6-(-2x)\,dx$

$= \int_{-2}^{0} 3x+6\,dx = \frac{3}{2}x^2+6x\Big|_{-2}^{0} = 0-\left(\frac{-93}{8}\right) = \frac{93}{8}$

region 2: $\int_0^2 x+6-x^3\,dx = \frac{1}{2}x^2+6x-\frac{1}{4}x^4\Big|_0^2 = 2+12-4 = 10$

$10+\frac{93}{8} = \frac{173}{8}$ or $21.625$

4. a. $\int \frac{1}{1+t}\,dt = \ln(1+t)+C$ $\quad f(0)=1$

$\cancel{\ln(1+0)}^{0} + C = 1 \quad C=1$

$\ln(1+t)+1$

b. $\int xe^{x^2}dx = \frac{1}{2}e^{x^2}+C$ $\quad u=x^2$, $du=2x\,dx$, $\frac{1}{2}du = x\,dx$ $\quad \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u+C = \frac{1}{2}e^{x^2}+C$

c. $\int \frac{e^t}{3+e^t}\,dt \quad \ln(3+e^t)+C$ $\quad u=3+e^t$, $du=e^t\,dt$ $\quad \int \frac{1}{u}\,du = \ln u + C = \ln(3+e^t)+C$

d. $\int \frac{4}{x\ln x}\,dx$ $\quad u=\ln x$, $du=\frac{1}{x}dx$ $\quad 4\int \frac{1}{u}\,du = 4\ln u + C = 4\ln(\ln x)+C$

e. $\int x\sqrt{x+1}\,dx$ $\quad u=\sqrt{x+1}$, $u^2=x+1$, $x=u^2-1$, $dx=2u\,du$ $\quad \int (u^2-1)u\cdot 2u\,du = \int 2u^4-2u^2\,du$

$\frac{2}{5}u^5-\frac{2}{3}u^3+C = \frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}+C$