174 Homework #12 Key

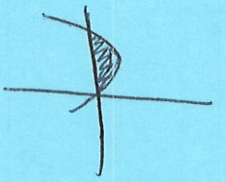
1a. $\frac{1}{3}\int_1^4 \frac{x^2+4}{x}\,dx = \frac{8\ln 2 + \frac{15}{2}}{3} = \frac{8}{3}\ln 2 + \frac{5}{2}$

b. $\frac{1}{2}\int_0^2 \frac{2x}{(1+x^2)^2}\,dx = \frac{4}{5}\cdot\frac{1}{2} = \frac{2}{5}$

c. $\frac{2}{\pi}\int_0^{\pi/2} e^{\sin t}\cos t\,dt = \frac{2}{\pi}(e-1)$

2. $500\cdot 40 + \int_0^{40} 40-y\,dy = 20{,}000 + 800 = 20{,}800$

3. a. $M = \int_0^2 2y - y^2\,dy = \frac{4}{3}$



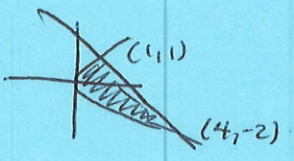
$M_x = \int_0^2 y(2y-y^2)\,dy = 4/3$

$M_y = \int_0^2 (2y-y^2)^2\,dy = \frac{16}{15}$

$\frac{M_y}{M} = \bar{x} = \frac{16}{15}\cdot\frac{3}{4} = \frac{4}{5}$ $\quad \left(\frac{4}{5}, 1\right)$

$\frac{M_x}{M} = \bar{y} = \frac{4}{3}\cdot\frac{3}{4} = 1$

b. $M = \int_{-2}^{1} (-y+2) - y^2\,dy = \frac{9}{2}$



$M_y = \int_{-2}^{1} (-y+2)^2 - (y^2)^2\,dy = \frac{72}{5}$

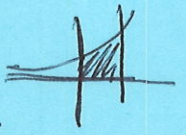
$M_x = \int_{-2}^{1} (-y+2-y^2)y\,dy = -9/4$

$\frac{M_y}{M} = \bar{x} = \frac{8}{\frac{9}{2}}\cdot\frac{2}{9} = \frac{16}{5}$

$\frac{M_x}{M} = \bar{y} = \frac{-\frac{9}{4}}{\frac{9}{2}}\cdot\frac{2}{9} = -\frac{1}{2}$

$\left(\frac{16}{5}, -\frac{1}{2}\right)$

c. $M = \int_0^1 e^x\,dx = e-1$



$M_x = \int_0^1 e^{2x}\,dx = \frac{e^2}{2} - \frac{1}{2}$

$M_y = \int_0^1 x e^x\,dx = 1$

$\bar{x} = \frac{M_y}{M} = \frac{1}{e-1}$

$\bar{y} = \frac{M_x}{M} = \frac{\frac{1}{2}(e^2-1)}{e-1} = \frac{1}{2}(e+1)$

$\left(\frac{1}{e-1}, \frac{1}{2}(e+1)\right)$

4. a $\pi \int_0^2 62.4 \cdot 20 \, dy (4-y) = 7488\pi$ bottom half

b. $\pi \int_0^4 62.4 \cdot 20 \, dy \, (4-y) = 9984\pi$

$\pi \int_2^4 62.4 \cdot 20 (4-y) dy = 2496\pi$ top half

5. $C = \int_0^{50} .6 + .008x \, dx + 100{,}000 = 140{,}000$

$\int_{50}^{200} .6 + .008x \, dx = 60$ or 60,000

6. $10 = k(\frac{1}{3})$ $k = 30$ $6in = \frac{1}{2}ft$ $4in = \frac{1}{3}ft$

$W = \int_0^{0.5} 30x \, dx = 3.75 ft \cdot lbs$

7. density $8 kg/m$

$W = \int_0^6 y(8y) \, dy = 576 \, N \cdot m$

8. $W = \int_{1/2}^1 1000(2\pi \cdot 1)(1-y) \, dy = 250 \, Nm$

9. $\pi(12)^2$

$W = \int_0^4 62.5 \cdot \pi (12)^2 (5-y) \, dy = 339{,}292 \, ft \cdot lbs$

10. $\frac{1}{3} \int_2^5 (x-3)^3 dx = \frac{5}{4}$

$(x-3)^3 = \frac{5}{4}$ $x - 3 = \sqrt[3]{\frac{5}{4}}$ $x = 3 + \sqrt[3]{\frac{5}{4}} \approx 4.07722$

11. $R = \int_0^4 9000\sqrt{1+2t} \, dt = 78{,}000$

12. $c \int_{-\infty}^{\infty} \frac{1}{1+x^2} dx = c(\frac{\pi}{2} - (-\pi/2)) = c\pi = 1$ $c = \frac{1}{\pi}$

13. a. $P(0 \le x \le 1) = \int_0^1 30x^2(1-x)^2 \, dx = 1$

d. $P(x \le \frac{1}{3}) = \int_0^{1/3} (30x^2)(1-x)^2 \, dx = \frac{17}{81}$

13b. $\int_0^1 x \cdot 30x^2(1-x)^2\,dx = \frac{1}{2}$

c. $\int_0^1 (x-\frac{1}{2})^2\, 30x^2(1-x)^2\,dx = \frac{1}{28}$

14. $\frac{1}{\sqrt{2\pi}}\int_{-1}^{1} e^{-x^2/2}\,dx$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\frac{1}{\sqrt{2\pi}}\int_{-1}^{1}\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{2^n n!}\,dn = \qquad e^{-x^2/2} = \sum_{n=0}^{\infty}\frac{(-x^2/2)^n}{n!} = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{2^n n!}$$

$$\frac{1}{\sqrt{2\pi}}\left[\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2^n(2n+1)n!}\right]\Bigg|_{-1}^{1} = \frac{1}{\sqrt{2\pi}}\left[\sum_{n=0}^{\infty}\frac{(-1)^n 1^{2n+1}}{2^n(2n+1)n!} - \frac{(-1)^n(-1)^{2n+1}}{2^n(2n+1)n!}\right] =$$

$n=5 \Rightarrow .682688$

$n=6 \Rightarrow .68269$

$.00001$ less