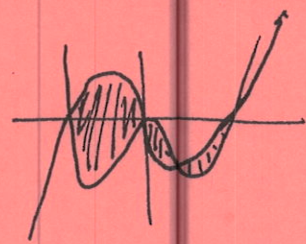
MTH 174 Homework #2 Key

1a. $f(x)=x^4-4x^2,\ g(x)=x^3-4x$



intersections: $-2, 0, 1, 2$

Region 1: Region 2: Region 3:

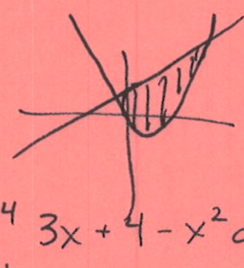
$$\int_{-2}^{0} x^3-4x-(x^4-4x^2)\,dx + \int_0^1 x^4-4x^2-(x^3-4x)\,dx + \int_1^2 x^3-4x-(x^4-4x^2)\,dx$$

$$\int_{-2}^{0} x^3-4x-x^4+4x^2\,dx + \int_0^1 x^4-4x^2-x^3+4x\,dx + \int_1^2 x^3-4x-x^4+4x^2\,dx$$

$$\tfrac{1}{4}x^4-2x^2-\tfrac{1}{5}x^5+\tfrac{4}{3}x^3\Big|_{-2}^{0} + \tfrac{1}{5}x^5-\tfrac{4}{3}x^3-\tfrac{1}{4}x^4+2x\Big|_0^1 + \tfrac{1}{4}x^4-2x^2-\tfrac{1}{5}x^5+\tfrac{4}{3}x^3\Big|_1^2$$

$$\frac{124}{15}+\frac{37}{60}+\frac{53}{60}=\boxed{\frac{293}{30}}$$

b. $y=x^2-2x,\ y=x+4$

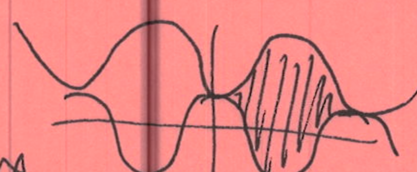


intersections: $-1, 4$

$$\int_{-1}^{4} x+4-(x^2-2x)\,dx = \int_{-1}^{4} x+4-x^2+2x\,dx = \int_{-1}^{4} 3x+4-x^2\,dx =$$

$$\tfrac{3}{2}x^2+4x-\tfrac{1}{3}x^3\Big|_{-1}^{4} = \boxed{\frac{125}{6}}$$

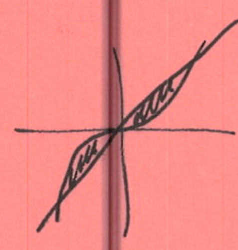
c. $y=\cos x,\ y=2-\cos x\quad [0,2\pi]$



$$\int_0^{2\pi}(2-\cos x)-\cos x\,dx = \int_0^{2\pi} 2-2\cos x\,dx$$

$$=2x-2\sin x\Big|_0^{2\pi} = 2(2\pi)-0=\boxed{4\pi}$$

d. $y=x^3,\ y=x$



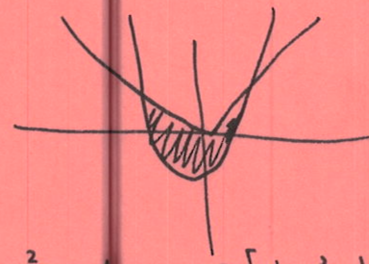
intersections: $-1, 0, 1$

$$\int_{-1}^{0} x^3-x\,dx + \int_0^1 x-x^3\,dx$$

use symmetry!

$$\tfrac{1}{4}x^4-\tfrac{1}{2}x^2\Big|_{-1}^{0} + \tfrac{1}{2}x^2-\tfrac{1}{4}x^4\Big|_0^1 = \tfrac{1}{4}+\tfrac{1}{4}=\boxed{\tfrac{1}{2}}$$

e. $y=|x|,\ y=x^2-2$



intersections: $-2, 2$ use symmetry

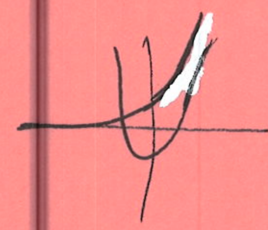
$$\int_{-2}^{0}|x|-(x^2-2)\,dx + \int_0^2 |x|-(x^2-2)\,dx =$$

$$\int_{-2}^{0}-x-(x^2-2)\,dx + \int_0^2 x-x^2+2\,dx = 2\int_0^2 x-x^2+2\,dx = 2\left[\tfrac{1}{2}x^2-\tfrac{1}{3}x^3+2x\right]_0^2 = \boxed{\frac{20}{3}}$$

1f. $y=e^x,\ y=x^2-1,\ x=-1, x=1$

$$\int_{-1}^{1} e^x-(x^2-1)\,dx = e^x-\frac{1}{3}x^3+x\Big|_{-1}^{1} =$$

$$e^1-\frac{1}{3}+1-e^{-1}+-\frac{1}{3}+1 = \boxed{e+\frac{1}{e}+\frac{4}{3}}$$



g. $x=1-y^2,\ x=y^2-1$

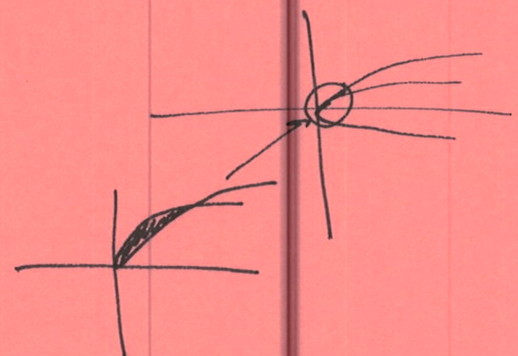
intersections: $-1, 1$

$$\int_{-1}^{1} 1-y^2-(y^2-1)\,dy = \int_{-1}^{1} 2-2y^2\,dy$$

$$2y-\frac{2}{3}y^3\Big|_{-1}^{1} = \boxed{\frac{8}{3}}$$

h. $x=y^4,\ y=\sqrt{2x}$ $\quad y=0$

$y=\pm x^{1/4}$



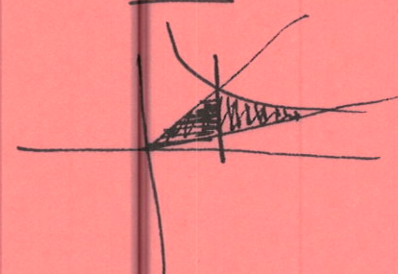
intersections: $0, \frac{1}{4}$

$$\int_0^{1/4} \sqrt[4]{x}-\sqrt{2x}\,dx$$

$$\int_0^{1/4} x^{1/4}-\sqrt{2}\,x^{1/2}\,dx = \frac{4}{5}x^{5/4}-\sqrt{2}\,\frac{2}{3}x^{3/2}\Big|_0^{1/4} = \frac{4}{5}\left(\frac{1}{4}\right)^{5/4}-\frac{2\sqrt{2}}{3}\left(\frac{1}{4}\right)^{3/2} =$$

$$\frac{\cancel{4}}{5}\cdot\frac{1}{\cancel{4\sqrt{2}}}\frac{\sqrt{2}}{\sqrt{2}}-\frac{\cancel{2}\sqrt{2}}{3}\cdot\frac{1}{\cancel{8}_4} = \frac{\sqrt{2}}{10}-\frac{\sqrt{2}}{12} = \sqrt{2}\left(\frac{1}{60}\right) = \boxed{\frac{\sqrt{2}}{60}}$$

i. $y=\frac{1}{x},\ y=x,\ y=\frac{1}{4}x,\ x>0$



$$\int_0^1 x-\frac{1}{4}x\,dx + \int_1^2 \frac{1}{x}-\frac{1}{4}x\,dx$$

$$\frac{1}{2}x^2-\frac{1}{8}x^2\Big|_0^1 + \ln x-\frac{1}{8}x^2\Big|_1^2 = \frac{3}{8}+\ln 2-\frac{3}{8} = \ln 2$$

j. $y=x^2e^{-x},\ y=xe^{-x}$

intersection: $0, 1$

$$\int_0^1 xe^{-x}-x^2e^{-x}\,dx = \int_0^1 (x-x^2)e^{-x}\,dx$$

$\pm$	u	dv
$+$	$x-x^2$	e^{-x}
$-$	$1-2x$	$-e^{-x}$
$+$	-2	e^{-x}
$-$	0	$-e^{-x}$

$$(x^2-x)e^{-x}+(2x-1)e^{-x}+2e^{-x}\Big|_0^1 = e^{-x}(x^2-x+2x-1+2)\Big|_0^1 =$$

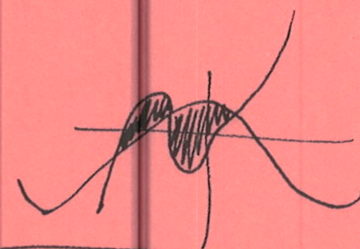
$$e^{-x}(x^2+x+1)\Big|_0^1 = e^{-1}(1+1+1)-e^0(0+0+1) = \boxed{\frac{3}{e}-1}$$

2a. $y = \frac{2}{1+x^4}$, $y = x^2$

intersections: $-1, 1$

$$\boxed{\int_{-1}^{1} \frac{2}{1+x^4} - x^2\, dx}$$

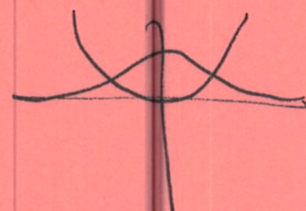
b. $y = \cos x$, $y = x + 2\sin^4 x$



intersections: -1.911917, -1.223676, 0.607945774

$$\boxed{\int_{-1.911917}^{-1.223676} x + 2\sin^4 x - \cos x\, dx + \int_{-1.223676}^{0.60794574} \cos x - x - 2\sin^4 x\, dx}$$

c. $y = e^{1-x^2}$, $y = x^4$



intersections: $-1, 1$

$$\boxed{\int_{-1}^{1} e^{1-x^2} - x^4\, dx}$$

3a. $y = 1 + 6x^{3/2}$ $[0,1]$

$y' = \cancel{3}\cancel{6} \cdot \frac{3}{\cancel{2}} x^{1/2} = 9x^{1/2}$

$$\int_0^1 \sqrt{1 + (9x^{1/2})^2}\, dx = \int_0^1 \sqrt{1 + 81x}\, dx \qquad u = 1 + 81x \quad 0 \to 1$$
$$du = 81\, dx \quad 1 \to 82$$

$$\frac{1}{81}\int_1^{82} u^{1/2}\, du = \frac{1}{81} \cdot \frac{2}{3} u^{3/2} \Big|_1^{82} = \boxed{\frac{2}{243}\left(82^{3/2} - 1\right)}$$

b. $y = 1 - e^{-x}$

$y' = e^{-x}$

$$\int_0^2 \sqrt{1 + e^{-2x}}\, dx \approx \boxed{2.22}$$

c. $y = \frac{x^3}{3} + \frac{1}{4x}$ $[1,2]$

$y' = \frac{3x^2}{3} + \frac{1}{4}(-1)x^{-2} = x^2 - \frac{x^{-2}}{4}$

$$\int_1^2 \sqrt{1 + \left(x^2 - \frac{x^{-2}}{4}\right)^2}\, dx = \int_1^2 \sqrt{1 + x^4 - \frac{1}{2} + \frac{x^{-4}}{16}}\, dx = \int_1^2 \sqrt{x^4 + \frac{1}{2} + \frac{x^{-4}}{16}}\, dx = \int_1^2 \sqrt{\left(x^2 + \frac{x^{-2}}{4}\right)^2}\, dx$$

$$= \int_1^2 x^2 + \frac{x^{-2}}{4}\, dx = \frac{1}{3}x^3 - \frac{1}{4x} \Big|_1^2 = \boxed{\frac{59}{24}}$$

d. $y = x\sin x$ $[0, 2\pi]$

$y' = \sin x + x\cos x$

$$\int_0^{2\pi} \sqrt{1 + (\sin x + x\cos x)^2}\, dx = \int_0^{2\pi} \sqrt{1 + \sin^2 x + 2x\sin x\cos x + x^2\cos^2 x}\, dx \approx 15.37$$

3e. $y = \ln(\sec x) \quad [0, \pi/4]$

$y' = \frac{\sec x \tan x}{\sec x} = \tan x$

$\int_0^{\pi/4} \sqrt{1 + \tan^2 x}\, dx = \int_0^{\pi/4} \sqrt{\sec^2 x}\, dx = \int_0^{\pi/4} \sec x\, dx = \ln|\sec x + \tan x| \Big|_0^{\pi/4} =$

$\ln|\sqrt{2}+1| - \cancel{\ln|1+0|}^{\,0} = \boxed{\ln|\sqrt{2}+1|}$

f. $y = \sqrt[3]{x} \quad [0,1]$

$y' = \frac{1}{3}x^{-2/3}$

$\int_0^1 \sqrt{1 + (\frac{1}{3}x^{-2/3})^2}\, dx = \int_0^1 \sqrt{1 + \frac{1}{9}x^{-4/3}}\, dx \approx \boxed{1.547}$

(integral is improper, try using a small non-zero boundary like 10^{-7})

g. $y = \sqrt{x - x^2} + \arcsin x \quad [0,1]$

$y' = \frac{1}{2}(x - x^2)^{-1/2}(1 - 2x) + \frac{1}{\sqrt{1-x^2}}$

$\int_0^1 \sqrt{1 + (\frac{1}{2}(x-x^2)^{-1/2}(1-2x) + (1-x^2)^{-1/2})^2}\, dx \approx \boxed{1.296}$

h. $y = e^{-x^2} \quad [-1,1]$

$y' = -2xe^{-x^2}$

$\int_{-1}^1 \sqrt{1 + (-2xe^{-x^2})^2}\, dx = \int_{-1}^1 \sqrt{1 + 4x^2e^{-2x^2}}\, dx \approx \boxed{2.409}$

4a. $\int x \cos 5x\, dx$

$u = x \qquad dv = \cos 5x\, dx$

$dx = du \qquad v = \frac{1}{5}\sin 5x$

$\frac{1}{5}x \sin 5x - \int \frac{1}{5}\sin 5x\, dx$

$= \boxed{\frac{1}{5}x \sin 5x + \frac{1}{25}\cos 5x + C}$

b. $\int \ln^2 x\, dx$

$u = \ln^2 x \qquad dv = dx$

$du = \frac{2\ln x}{x}\, dx \qquad v = x$

$x\ln^2 x - \int 2\ln x\, dx$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}\, dx \qquad v = x$

$x\ln^2 x - 2[x\ln x - \int dx]$

$= \boxed{x\ln^2 x - 2x\ln x + x + C}$

c. $\int t \cosh t\, dt$

$u = t \qquad dv = \cosh t\, dt$

$du = dt \qquad v = \sinh t$

$t\sinh t - \int \sinh t\, dt = \boxed{t\sinh t - \cosh t + C}$

4d. $\int x^3 e^{-x^2}\,dx$ $\quad u = x^2 \quad du = 2x\,dx \quad dv = xe^{-x^2}dx \quad v = -\frac{1}{2}e^{-x^2}$

$-\frac{1}{2}x^2e^{-x^2} + \frac{1}{\cancel{2}}\cdot\cancel{2}\int xe^{-x^2}dx = \boxed{-\frac{1}{2}x^2e^{-x^2} - \frac{1}{2}e^{-x^2} + C}$

e. $\int s2^s\,ds$ $\quad u = s \quad du = ds \quad dv = 2^s ds \quad v = \frac{1}{\ln 2}2^s$

$\frac{s2^s}{\ln 2} - \frac{1}{\ln 2}\int 2^s ds = \boxed{\frac{s2^s}{\ln 2} - \frac{1}{(\ln 2)^2}2^s + C}$

f. $\int \arctan 4t\,dt$ $\quad u = \arctan 4t \quad du = \frac{4}{1+16t^2} \quad dv = dt \quad v = t$

$t\arctan 4t - \int \frac{4t}{1+16t^2}dt$

$= t\arctan 4t - \frac{4}{32}\ln|1+16t^2| + C$ $\quad u = 1+16t^2 \quad du = 32t\,dt$

$\boxed{t\arctan 4t - \frac{1}{8}\ln|1+16t^2| + C}$

g. $\int (x^2+1)e^{-x}dx$ $\quad u = x^2+1 \quad du = 2x \quad dv = e^{-x}dx \quad v = -e^{-x}$

$-(x^2+1)e^{-x} + 2\int xe^{-x}dx = -(x^2+1)e^{-x} + 2[-xe^{-x} + \int e^{-x}dx]$

$u = x \quad du = dx \quad dv = e^{-x}dx \quad v = -e^{-x}$

$= \boxed{-(x^2+1)e^{-x} - 2xe^{-x} - 2e^{-x} + C}$

h. $\int_0^1 \frac{r^3}{\sqrt{4+r^2}}dr$ $\quad u = r^2 \quad du = 2r\,dr \quad dv = \frac{r}{\sqrt{4+r^2}}dr \quad v = (4+r^2)^{1/2}$

$w = 4+r^2 \quad dw = 2r\,dr \quad \int \frac{1}{2}w^{-1/2}dw = w^{1/2}$

$r^2\sqrt{4+r^2} - \int 2r(4+r^2)^{1/2}dr$

$= \boxed{r^2\sqrt{4+r^2} - \frac{2}{3}(4+r^2)^{3/2} + C}\Big|_0^1 = 1(\sqrt{5}) - \frac{2}{3}(5\sqrt{5}) - 0 + \frac{2}{3}4\sqrt{4} = \boxed{\frac{7\sqrt{5}}{3} + \frac{16}{3}}$

$w = (4+r^2) \quad dw = 2r\,dr \quad \int w^{1/2}dw$

i. $\int_0^\pi e^{\cos t}\sin 2t\,dt$ $\quad w = \cos t \quad dw = -\sin t\,dt \quad 0 \to 1 \quad \pi \to -1$

$= \int_0^\pi e^{\cos t}\, 2\sin t\cos t$

$= -2\int_1^{-1} we^w\,dw$ $\quad u = w \quad du = dw \quad dv = e^w dw \quad v = e^w$

$-2[we^w|_1^{-1} + \int_{-1}^1 e^w dw] = -2[we^w - e^w]_1^{-1} = 2[we^w - e^w]_{-1}^1 =$

$2[\cancel{1e^1} - \cancel{e^1} - (-1)e^{-1} + e^{-1}] = 2[2e^{-1}] = \boxed{\frac{4}{e}}$

5. $\int x^n \ln x \, dx$

$u = \ln x \qquad dv = x^n$

$du = \frac{1}{x} \qquad v = \frac{1}{n+1} x^{n+1}$

$$\frac{\ln x \cdot x^{n+1}}{n+1} - \int \frac{1}{n+1} \cdot \frac{1}{x} \cdot x^{n+1} \, dx =$$

$$\frac{x^{n+1} \ln x}{n+1} - \frac{1}{n+1} \int x^n \, dx = \frac{x^{n+1} \cdot \ln x}{n+1} - \frac{1}{(n+1)^2} x^{n+1} + C$$

equivalent to formula given

6. a.

±	u	dv
+	z^3	e^z
−	$3z^2$	e^z
+	$6z$	e^z
−	6	e^z
+	0	e^z

$\int z^3 e^z \, dz =$

$$\boxed{z^3 e^z - 3z^2 e^z + 6z e^z - 6 e^z + C}$$

b.

±	u	dv
+	t^4	$\sinh t$
−	$4t^3$	$\cosh t$
+	$12t^2$	$\sinh t$
−	$24t$	$\cosh t$
+	24	$\sinh t$
−	0	$\cosh t$

$\int t^4 \sinh t \, dt =$

$$\boxed{t^4 \cosh t - 4t^3 \sinh t + 12t^2 \cosh t - 24t \sinh t + 24 \cosh t + C}$$

c.

±	u	dv
+	p^5	$\cos p$
−	$5p^4$	$\sin p$
+	$20p^3$	$-\cos p$
−	$60p^2$	$-\sin p$
+	$120p$	$\cos p$
−	120	$\sin p$
+	0	$-\cos p$

$\int p^5 \cos p \, dp =$

$$\boxed{p^5 \sin p + 5p^4 \cos p - 20p^3 \sin p - 60p^2 \cos p + 120p \sin p + 120 \cos p + C}$$

7a. $\int e^{2\theta} \sin 3\theta \, d\theta$

$u = \sin 3\theta \qquad dv = e^{2\theta} d\theta$

$du = 3\cos 3\theta \, d\theta \qquad v = \frac{1}{2} e^{2\theta}$

$\frac{1}{2} e^{2\theta} \sin 3\theta - \frac{3}{2} \int e^{2\theta} \cos 3\theta \, d\theta$

$u = \cos 3\theta \qquad dv = e^{2\theta}$

$du = -3 \sin 3\theta \, d\theta \qquad v = \frac{1}{2} e^{2\theta}$

7a cont'd

$$\int e^{2\theta}\sin 3\theta\, d\theta = \frac{1}{2}e^{2\theta}\sin 3\theta - \frac{3}{4}e^{2\theta}\cos 3\theta - \frac{9}{4}\int e^{2\theta}\sin 3\theta\, d\theta$$

$$+\frac{9}{4}\int e^{2\theta}\sin 3\theta\, d\theta \qquad\qquad + \frac{9}{4}\int e^{2\theta}\sin 3\theta\, d\theta$$

$$\frac{13}{4}\int e^{2\theta}\sin 3\theta\, d\theta = \frac{1}{2}e^{2\theta}\sin 3\theta - \frac{3}{4}e^{2\theta}\cos 3\theta + C$$

$$\Rightarrow \int e^{2\theta}\sin 3\theta\, d\theta = \boxed{\frac{2}{13}e^{2\theta}\sin 3\theta - \frac{3}{13}e^{2\theta}\cos 3\theta + C}$$

7b. $\int \sec^3 d\theta$

$u = \sec\theta \qquad dv = \sec^2\theta\, d\theta$

$du = \sec\theta\tan\theta \qquad v = \tan\theta$

$\sec\theta\tan\theta - \int \sec\theta\tan^2\theta\, d\theta =$

$\sec\theta\tan\theta - \int \sec\theta(\sec^2\theta - 1)\, d\theta =$

$\sec\theta\tan\theta - \int \sec^3\theta\, d\theta + \int \sec\theta\, d\theta$

$\sec\theta\tan\theta - \int \sec^3\theta\, d\theta + \ln|\sec\theta + \tan\theta|$

$$\int \sec^3\theta\, d\theta = \sec\theta\tan\theta - \int\sec^3\theta\, d\theta + \ln|\sec\theta + \tan\theta|$$

$$+\int \sec^3\theta\, d\theta \qquad\qquad + \int \sec^3\theta\, d\theta$$

$$2\int \sec^3 d\theta = \sec\theta\tan\theta + \ln|\sec\theta + \tan\theta| + C$$

$$\int \sec^3 d\theta = \boxed{\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta| + C}$$