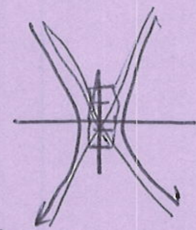
MTH 174 Homework #5 Key

1a. $4x^2-y^2-4x-3=0$

$4(x^2-x+\frac{1}{4})-y^2=3+1$

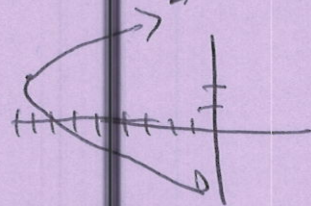
$\frac{4(x-\frac{1}{2})^2}{4}-\frac{y^2}{4}=\frac{4}{4}$

$(x-\frac{1}{2})^2-\frac{y^2}{4}=1$ hyperbola



b. $y^2-4y+4=x+5+4$ parabola

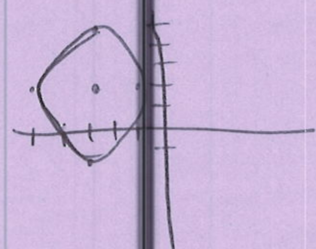
$(y-2)^2=x+9$



c. $9(x+3)^2=36-4(y-2)^2$

$\frac{9(x+3)^2}{36}+\frac{4(y-2)^2}{36}=\frac{36}{36}$ ellipse

$\frac{(x+3)^2}{4}+\frac{(y-2)^2}{9}=1$

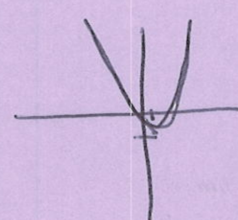


d. $25x^2-10x-200y-119=0$

$25(x^2-\frac{2}{5}x+\frac{1}{25})=200y+120$

$25(x-\frac{1}{5})^2=200(y+\frac{3}{5})$ parabola

$(x-\frac{1}{5})^2=8(y+\frac{3}{5})$

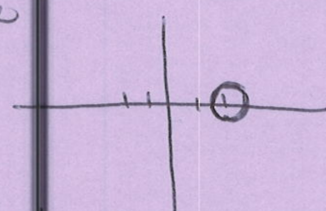


e. $9x^2+9y^2-36x+6y+34=0$

$9(x^2-4x+4)+9(y^2+\frac{2}{3}x+\frac{1}{9})=-34+36+1$

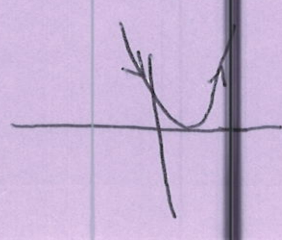
$\frac{9(x-2)^2}{3}+\frac{9(y+\frac{1}{3})^2}{3}=\frac{3}{3}$ ellipse

$\frac{(x-2)^2}{(1/3)}+\frac{(y+\frac{1}{3})^2}{(1/3)}=1$



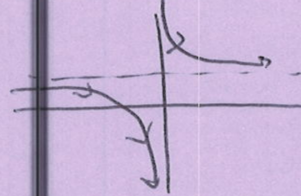
2a. $x=t+1,\ y=t^2$

$x-1=t \Rightarrow y=(x-1)^2$



2b. $x = t-1 \quad y = \frac{t}{t-1}$

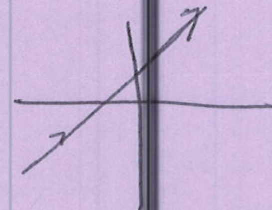
$x+1 = t \rightarrow y = \frac{x+1}{x} = 1 + \frac{1}{x}$



c. $x = \tan^2 t \quad y = \sec^2 t$

$1 + x = 1 + \tan^2 t = \sec^2 t = y$

$y = x+1$

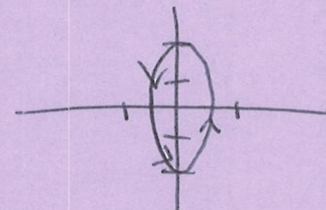


d. $x = \frac{1}{2}\cos\theta, \; y = 2\sin\theta$

$2x = \cos\theta \quad \frac{y}{2} = \sin\theta$

$4x^2 = \cos^2\theta \quad \frac{y^2}{4} = \sin^2\theta$

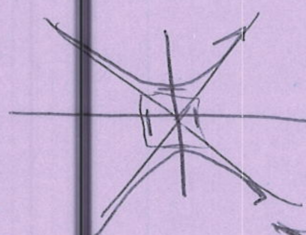
$\cos^2\theta + \sin^2\theta = 1 = 4x^2 + \frac{y^2}{4} \rightarrow \frac{x^2}{(1/4)} + \frac{y^2}{4} = 1$



e. $x = \sinh t, \; y = \cosh t$

$\cosh^2 t - \sinh^2 t = 1$

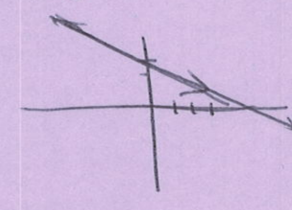
$y^2 - x^2 = 1$



f. $x = 3-2t \quad y = 2+3t$

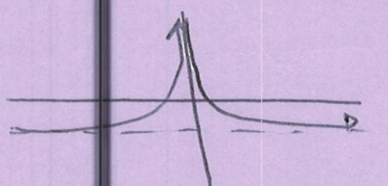
$\frac{x-3}{-2} = t = \frac{y-2}{3} \rightarrow 3x - 9 = -2y + 4$

$\frac{3x-13}{-2} = y \rightarrow y = -\frac{3}{2}x + \frac{13}{2}$



g. $x = e^{-t}, \; y = e^{2t} - 1$

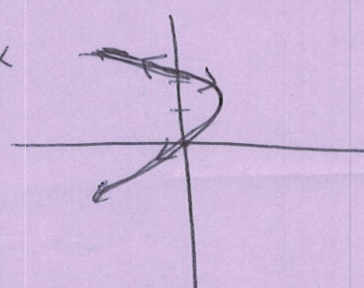
$\frac{1}{x} = e^t \rightarrow y = \left(\frac{1}{x}\right)^2 - 1 \rightarrow \frac{1}{x^2} - 1 = y$



h. $x = \cos^2 t \quad y = 1 - \sin t$

$x = 1 - \sin^2 t \quad 1 - y = \sin t$

$x = 1 - (1-y)^2 = 1 - (1 - 2y + y^2) = 1 - 1 + 2y - y^2 = 2y - y^2 = x$



3. a. $(1,4), (5,-2)$

$\langle -2-4, 5-1 \rangle = \langle -6, 4 \rangle$

$x = 1 - 6t, \; y = 4t + 4$

b. $(-3,1), \; r = 3$

$x = 3\cos t - 3, \; y = 3\sin t + 1$

4a. $x = t+1$ $y = t^2$

$\frac{dx}{dt} = 1$ $\frac{dy}{dt} = 2t$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{1} = 2t$

never vertical
horizontal at $t=0$
(1,0)

b. $x = t-1$, $y = \frac{t}{t-1}$

$\frac{dx}{dt} = 1$ $\frac{dy}{dt} = \frac{1(t-1) - 1(t)}{(t-1)^2} = \frac{-1}{(t-1)^2}$ $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-1}{(t-1)^2}$

vertical at $t=1$
never horizontal
(0, undefined)
not in domain

c. $x = \tan^2\theta$, $y = \sec^2\theta$

$\frac{dx}{d\theta} = 2\tan\theta\sec^2\theta$ $\frac{dy}{d\theta} = 2\sec\theta\sec\theta\tan\theta$ $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2\sec^2\theta\tan\theta}{2\tan\theta\sec^2\theta} = 1$

never vertical or horizontal

d. $x = \frac{1}{2}\cos\theta$, $y = 2\sin\theta$

$\frac{dx}{d\theta} = -\frac{1}{2}\sin\theta$ $\frac{dy}{d\theta} = 2\cos\theta$

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2\cos\theta}{-\frac{1}{2}\sin\theta} = -4\cot\theta$

vertical multiples of π
horizontal at odd multiples of $\pi/2$

e. $x = \sinh t$ $y = \cosh t$

$\frac{dx}{dt} = \cosh t$ $\frac{dy}{dt} = \sinh t$

$\frac{dy}{dx} = \frac{\sinh t}{\cosh t} = \tanh t$

horizontal at $t=0$
(0,1)

f. $x = 3-2t$, $y = 2+3t$

$\frac{dx}{dt} = -2$, $\frac{dy}{dt} = 3$

$\frac{dy}{dx} = \frac{3}{-2}$ never horizontal or vertical

g. $x = e^{-t}$, $y = e^{2t} - 1$

$\frac{dx}{dt} = -e^{-t}$ $\frac{dy}{dt} = 2e^{2t}$

$\frac{dy}{dx} = \frac{2e^{2t}}{-e^{-t}} = -2e^{3t}$

horizontal
never zero ($\to -\infty$)
never vertical

h. $x = \cos^2 t$, $y = 1 - \sin t$

$\frac{dx}{dt} = -2\cos t\sin t$ $\frac{dy}{dt} = -\cos t$

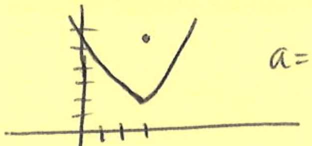
$\frac{dy}{dx} = \frac{-\cos t}{-2\cos t\sin t} = \frac{1}{2}\csc t$ vertical at multiples of π
horizontal never

5a. = i b = iii c = iv. d = ii

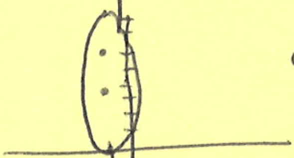
6a. $\vec{r}(t) = t\hat{i} + (4-t)\hat{j}$

b. $\vec{r}(t) = t\hat{i} + (4-t^2)\hat{j}$

c. $\vec{r}(t) = 5\cos t\,\hat{i} + 5\sin t\,\hat{j}$

7a. 

$a=4$ $\quad (x-3)^2 = 16(y-2)$

b. 

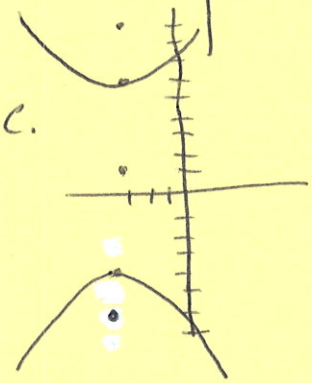
$a=4,\ c=2$

$2^2+b^2=4^2$

$b^2=16-4=12$

$b=2\sqrt{3}$

$$\frac{(x+1)^2}{12}+\frac{(y-4)^2}{16}=1$$

c. 

Center $(-3, 1)$

$a=5$

$c=8$

$8^2 \pm b^2 \neq 5^2$

$64-25=b^2$

$b=\sqrt{39}$

$$\frac{6-(-4)}{2}=\frac{10}{2}=5$$

$$\frac{(y-1)^2}{25}-\frac{(x+3)^2}{39}=1$$