MTH 174, Quiz #10, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Find the slope of the tangent line of $r = 3(1 - \cos\theta)$ at $\theta = \frac{\pi}{2}$.

$\frac{dr}{d\theta} = 3\sin\theta$

$\frac{dy}{dx} = \frac{3\sin\theta \cdot \sin\theta + 3(1-\cos\theta)\cos\theta}{3\sin\theta\cos\theta - 3(1-\cos\theta)\sin\theta} = \frac{3\sin^2\theta + 3 - 3\cos^2\theta}{3\sin\theta\cos\theta - 3 + 3\sin\theta\cos\theta}$

$= \frac{3(\sin^2\theta - \cos^2\theta + 1)}{6\sin\theta\cos\theta - 3} = \frac{\sin^2\theta - \cos^2\theta + 1}{2\sin\theta\cos\theta - 1} = \frac{1-0+1}{0-1} = \frac{2}{-1} = -2$

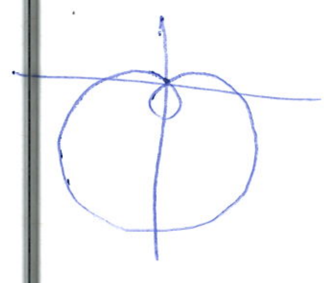
2. Find the area of the inner loop of $r = 4 - 6\sin\theta$.

$4 = 6\sin\theta$

$\frac{2}{3} = \frac{4}{6} = \sin\theta$

$\theta = \sin^{-1}\left(\frac{2}{3}\right) \approx .7297$

and $\pi - .7297 = 2.4119$



$\frac{1}{2}\int_{.7297}^{2.4119} (4 - 6\sin\theta)^2\, d\theta = \frac{1}{2}\int_{.7297}^{2.4119} 16 - 48\sin\theta + 36\sin^2\theta\, d\theta$

$= \frac{1}{2}\int_{.7297}^{2.4119} 16 - 48\sin\theta + 18 - 18\cos 2\theta\, d\theta = \frac{1}{2}\left[16\theta - 48\cos\theta + 18\theta - 9\sin 2\theta\right]_{.7297}^{2.4119}$

≈ 1.7635

3. Find the common interior of $r = 4\sin\theta, r = 2$.

$2 = 4\sin\theta$

$\frac{1}{2} = \sin\theta$

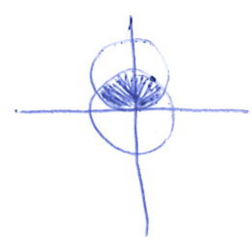
$\pi/6, 5\pi/6$

$\cancel{2}\left[\cancel{\tfrac{1}{2}}\int_0^{\pi/6} (4\sin\theta)^2\, d\theta + \cancel{\tfrac{1}{2}}\int_{\pi/6}^{\pi/2} 2^2\, d\theta\right]$

$\int_0^{\pi/6} 16\sin^2\theta\, d\theta + \int_{\pi/6}^{\pi/2} 4\, d\theta =$

$\int_0^{\pi/6} 8 - 8\cos 2\theta\, d\theta + \int_{\pi/6}^{\pi/2} 4\, d\theta$

$\left[8\theta - 4\sin 2\theta\right|_0^{\pi/6} + 4\theta\Big|_{\pi/6}^{\pi/2}$



$8(\pi/6) - 4\left(\frac{\sqrt{3}}{2}\right) + 4(\pi/2) - 4(\pi/6)$

$= \boxed{\frac{8\pi}{3} - 2\sqrt{3}}$

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$