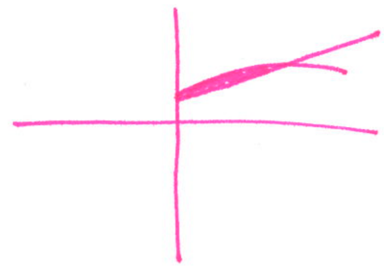
MTH 174, Quiz #2, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Find the area between $f(x) = \sqrt{x} + 3$ and $g(x) = \frac{1}{2}x + 3$. Sketch the region.

$$\sqrt{x} + \cancel{3} = \frac{1}{2}x + \cancel{3}$$
$$(\sqrt{x})^2 = \left(\frac{1}{2}x\right)^2$$
$$x = \frac{1}{4}x^2$$
$$4x = x^2$$
$$0 = x^2 - 4x = x(x-4)$$
$$x = 0, x = 4$$



$$\int_0^4 (\sqrt{x} + 3) - \left(\frac{1}{2}x + 3\right) dx =$$
$$\int_0^4 \sqrt{x} - \frac{1}{2}x\, dx =$$
$$\frac{2}{3}x^{3/2} - \frac{1}{4}x^2 \Big|_0^4 = \frac{2}{3}(4)^{3/2} - \frac{1}{4}(4)^2 = \frac{2}{3}(8) - 4 = \frac{16}{3} - 4 = \boxed{\frac{4}{3}}$$

2. Find the length of arc on $[1,27]$ for the function $y = \frac{3}{2}x^{2/3} + 4$. Give an exact solution.

$$f'(x) = x^{-1/3}$$
$$\int_1^{27} \sqrt{1 + \frac{1}{x^{2/3}}}\, dx =$$
$$\int_1^{27} \sqrt{\frac{x^{2/3} + 1}{x^{2/3}}}\, dx =$$
$$\int_1^{27} \frac{1}{x^{1/3}} \sqrt{x^{2/3} + 1}\, dx = \int_1^{27} x^{-1/3} \sqrt{x^{2/3} + 1}\, dx$$

$u = x^{2/3} + 1$

$du = \frac{2}{3}x^{-1/3}\,dx$

$\frac{3}{2}du = x^{-1/3}\,dx$

$$\frac{3}{2}\int_2^{10} u^{1/2}\, du = \frac{3}{2}\cdot\frac{2}{3}\cdot u^{3/2} \Big|_2^{10} = 10^{3/2} - 2^{3/2}$$