MTH 277 Homework #2 Key

1a. $\vec{AB} = \langle 1, -5, 4 \rangle$

$$\boxed{\vec{r}(t) = \underbrace{(t+2)}_{x}\hat{\imath} + \underbrace{(-5t+4)}_{y}\hat{\jmath} + \underbrace{(4t-3)}_{z}\hat{k}}$$

b. $\vec{AB} = \langle 11, -4, 1 \rangle$

$$\boxed{\frac{x+8}{11} = \frac{y-2}{-4} = \frac{z-4}{1}}$$

c. $\vec{v} = \langle 1, 2, 1 \rangle$

$$\boxed{\begin{gathered}\vec{r}(t) = (t+1)\hat{\imath} + (2t-1)\hat{\jmath} + (t+1)\hat{k} \\ \frac{x-1}{1} = \frac{y+1}{2} = \frac{z-1}{1}\end{gathered}}$$

2a. $\perp \langle 2, -1, 3 \rangle$

$$\boxed{2(x-5) - (y+3) + 3(z+4) = 0}$$

b. $\langle -2, 2, 0 \rangle$

$$\boxed{-2(x+6) + 2(y) + 0(z-8) = 0}$$

c. $\vec{AB} = \langle 1, 1, 4 \rangle$

$\vec{BC} = \langle -4, -5, -2 \rangle$

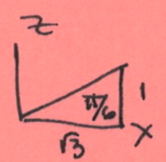
$$\vec{AB} \times \vec{BC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 4 \\ -4 & -5 & -2 \end{vmatrix} =$$

$$(-2+20)\hat{\imath} - (-2+16)\hat{\jmath} + (-5+4)\hat{k}$$

$$-18\hat{\imath} + 14\hat{\jmath} - 1\hat{k} \qquad \langle -18, 14, -1 \rangle$$

$$\boxed{\begin{aligned} -18(x-2) + 14(y-3) \\ -(z+2) = 0 \end{aligned}}$$

d.



$\langle \sqrt{3}, 0, 1 \rangle$

$\langle 0, 1, 0 \rangle$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 1 & 0 \\ \sqrt{3} & 0 & 1 \end{vmatrix} =$$

$$(1-0)\hat{\imath} - (0-0)\hat{\jmath} + (0-\sqrt{3})\hat{k}$$

$$\langle 1, 0, -\sqrt{3} \rangle$$

$$\boxed{1(x) + 0(y) - \sqrt{3}(z) = 0}$$

e. $\langle -2, 1, 1 \rangle$

$\langle -3, 4, -1 \rangle$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -2 & 1 & 1 \\ -3 & 4 & -1 \end{vmatrix} =$$

$P(1, 4, 0)$

$$(-1-4)\hat{\imath} - (2+3)\hat{\jmath} + (-8+3)\hat{k}$$

$$\langle -5, -5, -5 \rangle$$

$$\boxed{-5(x-1) - 5(y-4) - 5(z) = 0}$$

②

2.f $\langle -3,-1,-2\rangle \quad \langle 2,-3,1\rangle$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -3 & -1 & -2 \\ 2 & -3 & 1 \end{vmatrix} =$$

$$(-1-6)\hat{\imath} - (-3+4)\hat{\jmath} + (9+2)\hat{k}$$

$$\langle -7,-1,11\rangle$$

$$\boxed{-7(x-2)-(y-2)+11(z-1)=0}$$

3. $\langle 3,2,-1\rangle$

$\langle 1,-4,2\rangle$

$(0,0,-7)$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 2 & -1 \\ 1 & -4 & 2 \end{vmatrix} =$$

$$(4-4)\hat{\imath} - (6+1)\hat{\jmath} + (-12-2)\hat{k}$$

$$\langle 0,-7,-14\rangle$$

$z = 3x+2y-7$

$x-4y+2(3x+2y-7)=0$

$x-\cancel{4y}+6x+\cancel{4y}-14=0$

$7x=14$

$x=2$

$6+2y-z=7$

$2y-z=1$

$z=2y-1$

$2-4y+2z=0$

$-4y+2z=-2$

$4y-2z=2$

$0=0$

$x=2,\ y=-7t,\ z=2(-7t-1)$

$$\boxed{\vec{r}(t) = 2\hat{\imath} - 7t\hat{\jmath} + (-14t-2)\hat{k}}$$

$\vec{u}\cdot\vec{v} =$

$3-6-2=-5$

$\|\vec{u}\| = \sqrt{9+4+1} = \sqrt{14}$

$\|\vec{v}\| = \sqrt{1+16+4} = \sqrt{21}$

$$\cos^{-1}\left(\frac{-5}{\sqrt{14}\sqrt{21}}\right) = \boxed{\begin{array}{l}1.8667 \text{ radians} \\ 106.95^\circ \approx 107^\circ\end{array}}$$

4a. $Q=(0,0,5)$

$\vec{PQ} = \langle 2,8,-1\rangle$

$\vec{n} = \langle 2,1,1\rangle$

$\vec{PQ}\cdot\vec{n} = 4+8-1 = 11$

$\|n\| = \sqrt{4+1+1} = \sqrt{6}$

$$\boxed{D = \frac{11}{\sqrt{6}}}$$

b. $Q=(1,2,0)$

$\vec{PQ} = \langle 3,1,-3\rangle$

$\vec{v} = \langle -1,1,-2\rangle$

$$\vec{PQ}\times\vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 1 & -3 \\ -1 & 1 & -2 \end{vmatrix} =$$

$$(-2+3)\hat{\imath} - (-6-3)\hat{\jmath} + (3+1)\hat{k}$$

$\langle 1,9,4\rangle \qquad \|\vec{PQ}\times\vec{v}\| = \sqrt{1+81+16} = \sqrt{98}$

$\|\vec{v}\| = \sqrt{1+1+4} = \sqrt{6}$

$$\boxed{D = \frac{\sqrt{98}}{\sqrt{6}} = \frac{7}{\sqrt{3}}}$$

5a. D: $-2 \le t \le 2$

$\|\vec{r}(t)\| = \sqrt{\sqrt{(4-t^2)^2} + (t^2)^2 + (-6t)^2} = \sqrt{4-t^2+t^4+36t^2} = \sqrt{t^4+35t^2+4}$

b. D: $t \neq -1$

$\|r(t)\| = \sqrt{(t^{1/3})^2 + \left(\frac{1}{t+1}\right)^2 + (t+2)^2} = \sqrt{t^{2/3} + \frac{1}{(t+1)^2} + t^2+4t+4}$

c. D: $t > 0$

$\|r(t)\| = \sqrt{(\ln t - 1)^2 + t^2}$

d. D: $t \ge 0$

$\|r(t)\| = \sqrt{(1-t)^2 + (\sqrt{t})^2} = \sqrt{1-2t+t^2+t} = \sqrt{t^2-t+1}$

6a. $\vec{r}_1(t) = t\hat{\imath} + (4-t)\hat{\jmath}$ $\qquad x = 4-y$

$\vec{r}_2(t) = (4-t)\hat{\imath} + t\hat{\jmath}$

b. $\vec{r}_1(t) = 5\cos t\,\hat{\imath} + 5\sin t\,\hat{\jmath}$

$\vec{r}_2(t) = 5\sin t\,\hat{\imath} + 5\cos t\,\hat{\jmath}$

7. a. D: $\{(x,y) \mid x \in \mathbb{R}, y \in \mathbb{R}\}$

$f(0,0) = 4$, $f(2,3) = 4-4-9 = -9$ $\quad f(1,y) = 4-1-y^2 = 3-y^2$

$f(x,0) = 4-x^2$, $f(t,t^2) = 4-t^2-t^4$

b. D: $\{(x,y) \mid xy > 6\}$

$f(5,e) = \ln(5e-6)$, $f(e,1) = \ln(e-6)$ not defined $\quad f(1,y) = \ln(y-6)$

$f(x,0) = \ln(-6)$ not defined $\quad f(t,e^t) = \ln(te^t-6)$

8a. $z = x^2+y^2$, $x+y=0$ $\quad x = t$

$t = -y \rightarrow y = -t$ $\qquad \vec{r}(t) = t\hat{\imath} - t\hat{\jmath} + 2t^2\hat{k}$

$z = t^2 + (-t)^2 = 2t^2$

b. $z = \sqrt{x^2+y^2}$ $\quad z = 1+y$ $\qquad y = t$, $z = 1+t$ $\qquad x = \sqrt{1+2t}$

$\boxed{\vec{r}(t) = \sqrt{1+2t}\,\hat{\imath} + t\hat{\jmath} + (1+t)\hat{k}}$

$z^2 = x^2+y^2$

$(1+t)^2 = x^2+t^2 \rightarrow x^2 = 1+2t+\cancel{t^2}-\cancel{t^2}$

$= 1+2t$

answers will vary

8c. $x^2+y^2+z^2=4$, $x+z=2$ $x=1+\sin t$

$z=2-x = 2-(1+\sin t) = 1-\sin t$

$y^2 = 4-x^2-z^2 = 4-(1+\sin t)^2 - (1-\sin t)^2 =$

$4-(1+2\sin t+\sin^2 t)-(1-2\sin t+\sin^2 t)$

$=4-1-2\cancel{\sin t}-\sin^2 t-1+2\cancel{\sin t}-\sin^2 t$

$2-2\sin^2 t = 2(1-\sin^2 t) = 2\cos^2 t$

$y=\sqrt{2-2\sin^2 t} = \sqrt{2}\cos t$

$\boxed{\vec{r}(t) = (1+\sin t)\hat{\imath} + \sqrt{2}\cos t\,\hat{\jmath} + (1-\sin t)\hat{k}}$

d. $z=4x^2+y^2$, $y=x^2$ $x=t$, $y=t^2$

$z=4t^2+(t^2)^2 = 4t^2+t^4$

$\boxed{\vec{r}(t) = t\,\hat{\imath} + t^2\hat{\jmath} + (4t^2+t^4)\hat{k}}$

9a. $\vec{r}_1(t) = 3\cos t\,\hat{\imath} + 3\sin t\,\hat{\jmath}$

$\vec{r}_2(t) = 3(\cos(t+2\pi))\hat{\imath} + 3\sin(t+2\pi)\hat{\jmath}$

b. $\vec{r}_1(t) = 4\cos t\,\hat{\imath} + 3\sin t\,\hat{\jmath}$ $[0, \pi/2]$

$\vec{r}_2(t) = 4\cos(t+2\pi)\hat{\imath} + 3\sin(t+2\pi)\hat{\jmath}$

c. $\langle 1,0,0\rangle$ $\vec{r}_1(t) = t\,\hat{\imath}$

$\langle 0,0,1\rangle$ $\vec{r}_2(t) = t\hat{k} + 1\hat{\imath} = \hat{\imath} + t\hat{k}$ $[0,1]$

$\langle 0,1,0\rangle$ $\vec{r}_3(t) = 1\hat{\imath} + t\hat{\jmath} + 1\hat{k}$

d. $\vec{r}_1(t) = t\,\hat{\imath} + t^2\hat{\jmath}$ $[0,2]$

$\langle -2,0\rangle$ $\vec{r}_2(t) = (-2t-2)\hat{\imath} + 4\hat{\jmath}$

$\langle 0,-4\rangle$ $\vec{r}_3(t) = 0\hat{\imath} - 4t\,\hat{\jmath}$

e. $y=4\sec t$ $x=2\tan t$ $-56° \approx -.983$ radians

$\vec{r}(t) = 2\tan t\,\hat{\imath} + 4\sec t\,\hat{\jmath}$ $t\in[-1,1]$