MTH 277 Homework #3 Key

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1a. $x^2+y^2+z^2=10$ cylindrical: $r^2+z^2=10$ spherical $\rho^2=10$

$\to \rho=\sqrt{10}$

b. $x^2+y^2=9$ cylindrical $r^2=9$

$\to r=3$

spherical $\rho^2\sin^2\varphi=9$

$\to \rho\sin\varphi=3$

spherical $\to \rho=3\csc\varphi$

c. $x^2+y^2-3z^2=0$ cylindrical

$r^2-3z^2=0$

$r^2=3z^2$

$\to r=\sqrt{3}z$ or

$z=\frac{1}{\sqrt{3}}r$

spherical

$\cancel{\rho^2}\sin^2\varphi-3\cancel{\rho^2}\cos^2\varphi=0$

$\sin^2\varphi=3\cos^2\varphi$

$\tan^2\varphi=3 \to \tan\varphi=\sqrt{3}$

$\varphi=\pi/3$

d. $y=x^2$ cylindrical $\cancel{r}\sin\theta=r^{\cancel{2}}\cos^2\theta$

$\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta}=r$

$r=\tan\theta\sec\theta$

spherical -

$\cancel{\rho}\sin\varphi\sin\theta=\rho^{\cancel{2}}\sin^2\varphi\cos^2\theta$

$\frac{\sin\varphi\sin\theta}{\sin^2\varphi\cos\theta\cos\theta}=\rho$

$\rho=\csc\varphi\tan\theta\sec\theta$

e. $x=4$ cylindrical

$r\cos\theta=4$

$r=4\sec\theta$

spherical

$\rho\cos\theta\sin\varphi=4$

$\rho=4\sec\theta\csc\varphi$

2a. $r=2 \to x^2+y^2=4$

b. $z=r^2\cos^2\theta \to z=x^2$

c. $\rho=2\sec\varphi \to \rho\cos\varphi=2$

$\to$ $z=2$

d. $\rho=2 \to x^2+y^2+z^2=4$

e. $r=2\sin\theta \to r^2=2r\sin\theta \to x^2+y^2=2y$

f. $r=\frac{1}{2}z \to r^2=\frac{1}{4}z^2 \to x^2+y^2=\frac{1}{4}z^2 \to 4x^2+4y^2-z^2=0$

g. $\rho=4\csc\varphi\sec\theta \to \rho\sin\varphi\cos\theta=4 \to x=4$

h. $\varphi=\pi/6 \to \tan\varphi=\frac{1}{\sqrt{3}} \to \sin\varphi=\frac{1}{\sqrt{3}}\cos\varphi \to \rho\sin\varphi=\frac{1}{\sqrt{3}}\rho\cos\varphi$

$\to \rho^2\sin^2\varphi=\frac{1}{3}\rho^2\cos^2\varphi \to x^2+y^2=\frac{1}{3}z^2 \to 3x^2+3y^2-z^2=0$

3a. $\vec{r}(u,v) = u\hat{\imath} + v\hat{\jmath} + (6-u-v)\hat{k}$

$x = u, y = v$

$z = 6 - x - y$

b. $\vec{r}(u,v) = u\cos v\,\hat{\imath} + u\sin v\,\hat{\jmath} + 4\hat{k}$ $\quad u \in [0,3],\ v \in [0,2\pi]$

c. $\vec{r}(u,v) = 3\cos u \sin v\,\hat{\imath} + 2\sin u \sin v\,\hat{\jmath} + \cos v\,\hat{k}$ $\quad u \in [0,2\pi],\ v \in [0,\pi]$

d.=e. $\dfrac{4x^2 + y^2 = 16}{16} \rightarrow \dfrac{x^2}{4} + \dfrac{y^2}{16} = 1$ $\quad \vec{r}(u,v) = 2\cos u\,\hat{\imath} + 4\sin u\,\hat{\jmath} + v\hat{k}$

e.=f. $z = y$

$\vec{r}(u,v) = u\hat{\imath} + v\hat{\jmath} + v\hat{k}$

4a. $\vec{r}(u,v) = u\hat{\imath} + v\hat{\jmath} + \frac{v}{2}\hat{k} \rightarrow z = \frac{y}{2}$

b. $\vec{r}(u,v) = 2u\cos(v)\,\hat{\imath} + 2u\sin v\,\hat{\jmath} + \frac{1}{2}u^2\hat{k}$

$\dfrac{x^2 + y^2 = 4u^2}{8} \rightarrow \frac{1}{8}(x^2+y^2) = z$

c. $\vec{r}(u,v) = 3\cos v\cos u\,\hat{\imath} + 3\cos v \sin u\,\hat{\jmath} + 5\sin v\,\hat{k}$

$\dfrac{x^2}{9} + \dfrac{y^2}{9} + \dfrac{z^2}{25} = 1$

d. $\vec{r}(u,v) = 4\cos u\,\hat{\imath} + 4\sin u\,\hat{\jmath} + v\hat{k}$

$x^2 + y^2 = 16$

e. $\vec{r}(u,v) = u\hat{\imath} + v\hat{\jmath} + \sqrt{uv}\,\hat{k} \rightarrow z = \sqrt{xy}$

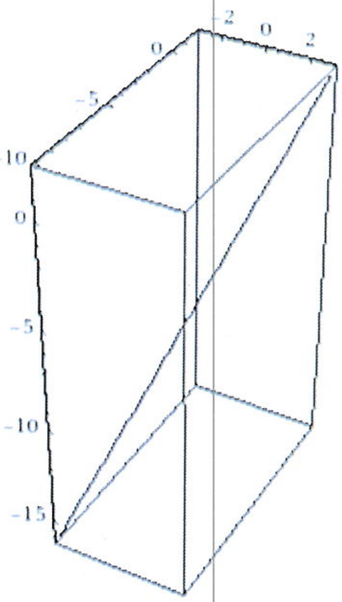
5. see attached

6a = v $\quad$ b = iv $\quad$ c = vi $\quad$ d = iii $\quad$ e = i $\quad$ f = ii

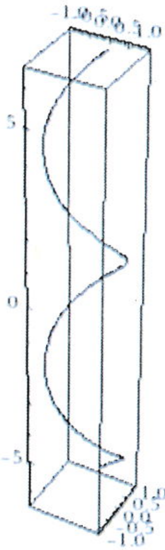
7a. $\displaystyle\lim_{(x,y)\to(2,1)} \frac{(x-y-1)(\sqrt{x-y}+1)}{(\sqrt{x-y}-1)(\sqrt{x-y}+1)} = \lim_{(x,y)\to(2,1)} \frac{(\cancel{x-y-1})(\sqrt{x-y}+1)}{\cancel{x-y-1}} = 2$

b. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = \lim_{x\to 0} \frac{x\cdot kx}{x^2+k^2x^2} = \lim_{x\to 0}\frac{\cancel{x^2}k}{\cancel{x^2}(1+k^2)} = \frac{k}{1+k^2}$ DNE $\quad y = kx$

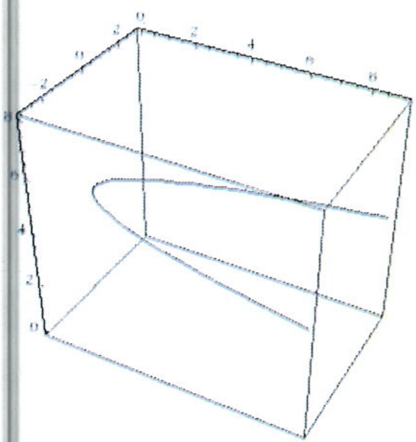
c. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{xy} = \lim_{x\to 0}\frac{x^2\cdot(k^2+1)}{x^2\cdot k} = \frac{k^2+1}{k}$ DNE $\quad y = kx$



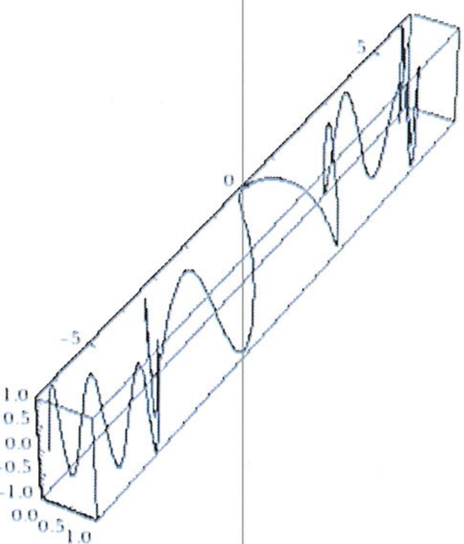

5a.




b.



c.




d.

7d. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y^2}{x^4+3y^4} = \lim\limits_{x\to 0} \dfrac{x^2k^2x^2}{x^4+3k^4x^4} = \lim\limits_{x\to 0} \dfrac{\cancel{x^4}k^2}{\cancel{x^4}(1+3k^4)}$ $\quad y = kx$

$= \dfrac{k^2}{1+3k^4}$ DNE

e. $\lim\limits_{(x,y)\to(0,0)} \dfrac{y^2\sin^2 x}{x^4+y^4} = \lim\limits_{x\to 0} \dfrac{k^2x^2\sin^2 x}{x^4+k^4x^4} =$ $\quad y = kx$

$\lim\limits_{x\to 0} \dfrac{\cancel{x^2}k^2\sin^2 x}{x^{\cancel{4}2}(1+k^4)} = \lim\limits_{x\to 0}\left(\overset{1}{\cancel{\dfrac{\sin^2 x}{x^2}}}\right)\left(\dfrac{k^2}{1+k^4}\right) = \dfrac{k^2}{1+k^4}$ DNE

f. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y^5}{2x^4+3y^{10}} = \lim\limits_{y\to 0} \dfrac{k^2y^5y^5}{2k^4y^{10}+3y^{10}} = \lim\limits_{y\to 0} \dfrac{k^2\cancel{y^{10}}}{\cancel{y^{10}}(2k^4+3)}$ $\quad x^4 = y^{10}$, $x = ky^{5/2}$

$= \lim\limits_{y\to 0} \dfrac{k^2}{2k^4+3} =$ DNE

g. $\lim\limits_{(x,y)\to(0,0)} \dfrac{3x^3+2\sqrt{y}}{x^2+y^2} = \lim\limits_{x\to 0} \dfrac{3x^3+2\sqrt{kx}}{x^2(1+k^2)} =$ $\quad y = kx$

$\lim\limits_{x\to 0} \dfrac{3x^3}{x^2(1+k^2)} + \lim\limits_{x\to 0} \dfrac{2\sqrt{k}\cdot\sqrt{x}}{x^2(1+k^2)} = \lim\limits_{x\to 0} \dfrac{3x}{1+k^2} + \lim\limits_{x\to 0} \dfrac{2\sqrt{k}}{x^{3/2}(1+k^2)}$

$= 0 +$ DNE $=$ DNE

h. $\lim\limits_{(x,y,z)\to(0,0,0)} \arctan\left[\dfrac{1}{x^2+y^2+z^2}\right] =$

$\lim\limits_{\rho\to 0} \arctan\left[\dfrac{1}{\rho^2}\right] = \pi/2$

i. $\lim\limits_{(x,y)\to(0,0)} \dfrac{(x-y)}{(\sqrt{x}-\sqrt{y})}\cdot\dfrac{(\sqrt{x}+\sqrt{y})}{(\sqrt{x}+\sqrt{y})} = \lim\limits_{(x,y)\to(0,0)} \dfrac{\cancel{(x-y)}(\sqrt{x}+\sqrt{y})}{\cancel{x-y}} = 0$

j. $\lim\limits_{(x,y)\to(0,0)} \dfrac{2x-y^2}{2x^2+y} = \lim\limits_{x\to 0} \dfrac{2x-k^2x^4}{2x^2+kx^2} = \lim\limits_{x\to 0} \dfrac{2\cancel{x}}{x^{\cancel{2}}(2+k)} + \lim\limits_{x\to 0} \dfrac{k^2x^{\cancel{4}2}}{\cancel{x^2}(2+k)} =$ $\quad y = x^2k$

$\lim\limits_{x\to 0} \dfrac{2}{x(2+k)} + \lim\limits_{x\to 0} \dfrac{k^2x^2}{2+k} =$ DNE $+ 0 =$ DNE

k. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y}{x^3+y^2} = \lim\limits_{y\to 0} \dfrac{k^2y^{4/3}y}{k^3y^2+y^2} = \lim\limits_{y\to 0} \dfrac{\cancel{y^2}(k^2y^{1/3})}{\cancel{y^2}(k^3+1)} =$ $\quad x^3 = y^2$, $x = ky^{2/3}$

$\lim\limits_{y\to 0} \dfrac{k^2\sqrt[3]{y}}{k^3+1} = 0$

7l. $\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^6+y^2} = \lim_{x\to0} \frac{x^3\cdot kx^3}{x^6+k^3x^6} = \lim_{x\to0} \frac{\cancel{x^6}k}{\cancel{x^6}(1+k^3)} =$ $\quad x^6=y^2$, $kx^3=y$

$\frac{k}{1+k^3}$ DNE

m. $\lim_{(x,y)\to(0,0)} \frac{x^2ye^y}{x^4+4y^2} = \lim_{x\to0} \frac{x^2\cdot kx^2e^{kx^2}}{x^4+4k^2y^{\cancel{x}}} = \lim_{x\to0} \frac{\cancel{x^4}ke^{kx^2}}{\cancel{x^4}(1+4k^2)}$ $\quad y=kx^2$

$= \lim_{x\to0} \frac{ke^{kx^2}}{1+4k^2} = \frac{k}{1+4k^2}$ DNE

n. $\lim_{(x,y)\to(0,0)} \frac{2x^3y^2}{x^6+y^4} = \lim_{x\to0} \frac{2x^3k^2x^3}{x^6+k^4x^6} = \lim_{x\to0} \frac{\cancel{x^6}(2k^2)}{\cancel{x^6}(1+k^4)}$ $\quad x^6=y^4$, $x^3=y^2$, $kx^{3/2}=y$

$= \frac{2k^2}{1+k^4}$ DNE

o. $\lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2+y^2+z^2} = \lim_{\rho\to0} \frac{\rho\cos\theta\sin\varphi\,\rho\sin\theta\sin\varphi\,\rho\cos\varphi}{\rho^2} =$

$\lim_{\rho\to0} \frac{\rho^{\cancel{3}}(\cos\theta\sin^2\varphi\sin\theta\cos\varphi)}{\cancel{\rho^2}} = 0$

p. $\lim_{(x,y,z)\to(0,0,0)} \frac{xy+yz+xz}{x^2+y^2+z^2} = \lim_{\rho\to0} \frac{\rho\cos\theta\sin\varphi\,\rho\sin\theta\sin\varphi + \rho\sin\theta\sin\varphi\,\rho\cos\varphi + \rho\cos\theta\sin\varphi\,\rho\cos\varphi}{\rho^2}$

$= \lim_{\rho\to0} \frac{\cancel{\rho^2}(\cos\theta\sin^2\varphi\sin\theta + \rho\sin\theta\sin\varphi\cos\varphi + \cos\theta\sin\varphi\cos\theta)}{\cancel{\rho^2}} =$

$\cos\theta\sin^2\varphi\sin\theta + \sin\theta\sin\varphi\cos\varphi + \cos\theta\sin\varphi\cos\theta$ DNE

8a = vii $\quad$ b = i $\quad$ c = viii $\quad$ d = vi $\quad$ e = v $\quad$ f = iii $\quad$ g = iv $\quad$ h = ii

9. a = iii $\quad$ b = v $\quad$ c = i $\quad$ d = iv $\quad$ e = ii $\quad$ f = vi

10. a = $\quad$ b = v $\quad$ c = vi $\quad$ d = i $\quad$ e = iv $\quad$ f = ii

11a. $x^2+y^2-4>0 \quad x^2+y>4$ continuous everywhere on domain

b. not continuous at $(0,0)$

c. $0 \le x^2+y^2+z^2 \le 1$ continuous on domain

d. $1+x-y \ge 0 \quad 1+x \ge y$ $\quad$ continuous on domain