277 Homework #6 Key

1a. $\int_0^1\int_0^2 (x+y)\,dy\,dx = \int_0^1 xy+\frac{1}{2}y^2\Big|_0^2 dx = \int_0^1 2x+2\,dx = x^2+2x\Big|_0^1 = \boxed{3}$

b. $\int_0^2\int_{3y^2-6y}^{2y-y^2} 3y\,dx\,dy = \int_0^2 3xy\Big|_{3y^2-6y}^{2y-y^2} dy = \int_0^2 3y(2y-y^2)-3y(3y^2-6y)\,dy$

$= \int_0^2 6y^2-3y^3-9y^3+18y^2\,dy = \int_0^2 24y^2-12y^3\,dy = 8y^3-3y^4\Big|_0^2 =$

$64-48 = \boxed{16}$

c. $\int_0^1\int_0^\infty \frac{x^2}{1+y^2}\,dy\,dx = \int_0^1 \lim_{b\to\infty} x^2\arctan y\Big|_0^b dx = \int_0^1 \frac{\pi}{2}x^2\,dx = \frac{\pi}{6}x^3\Big|_0^1 = \boxed{\frac{\pi}{6}}$

d. $\int_0^1\int_1^{\ln 4} \frac{\sinh x}{1+\sinh^2 y}\,dy\,dx = \int_0^1\int_1^{\ln 4}\frac{\sinh x}{\cosh^2 y}\,dy\,dx = \int_0^1\int_1^{\ln 4} \sinh x\cdot\operatorname{sech}^2 y\,dy\,dx$

$= \int_0^1 -\tanh y\cdot\sinh x\Big|_1^{\ln 4} dx = \int_0^1\left(-\frac{15}{17}+\tanh 1\right)\sinh x\,dx = \left(-\frac{15}{17}+\tanh 1\right)\cosh x\Big|_0^1$

$= \left(-\frac{15}{17}+\tanh 1\right)(\cosh 1-1)$

e. $\int_0^{\pi/2}\int_0^{1-\cos\theta} \sin\theta\cdot r\,dr\,d\theta = \int_0^{\pi/2}\frac{1}{2}r^2\Big|_0^{1-\cos\theta}\sin\theta\,d\theta = \frac{1}{2}\int_0^{\pi/2}(1-2\cos\theta+\cos^2\theta)\sin\theta\,d\theta$

$= \frac{1}{2}\int_0^{\pi/2} \sin\theta-2\sin\theta\cos\theta+\cos^2\theta\sin\theta\,d\theta = \frac{1}{2}\left[-\cos\theta-\frac{2}{2}\sin^2\theta-\frac{1}{3}\cos^3\theta\right|_0^{\pi/2}$

$= \frac{1}{2}\left[0-\cancel{1}-0+\cancel{1}+0+\frac{1}{3}\right] = \boxed{\frac{1}{6}}$

f. $\int_1^4\int_1^{\sqrt{x}} 2ye^{-x}\,dy\,dx = \int_1^4 y^2e^{-x}\Big|_1^{\sqrt{x}} dx = \int_1^4 xe^{-x}-e^{-x}\,dx = \int_1^4 e^{-x}(x-1)\,dx$

$u=x-1$ $\quad dv=e^{-x}dx$ $\quad -(x-1)e^{-x}+\int_1^4 e^{-x}dx = -(x-1)e^{-x}-e^{-x}\Big|_1^4 =$

$du=dx$ $\quad v=-e^{-x}$

$-(3)e^{-4}-e^{-4}+(0)e^{-1}+e^{-1} = \boxed{-4e^{-4}+e^{-1}}$

g. $\int_0^{\pi/4}\int_0^{\cos\theta} 3r^2\sin\theta\,dr\,d\theta = \int_0^{\pi/4} r^3\Big|_0^{\cos\theta}\sin\theta\,d\theta = \int_0^{\pi/4}\cos^3\theta\sin\theta\,d\theta =$

$-\frac{1}{4}\cos^4\theta\Big|_0^{\pi/4} = -\frac{1}{4}\left(\frac{1}{\sqrt{2}}\right)^4+\frac{1}{4}(1)^4 = \frac{1}{4}-\frac{1}{16} = \boxed{\frac{3}{16}}$

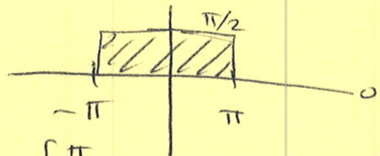
1h. $\int_0^4\int_0^{x/2} dy\,dx + \int_4^6\int_0^{6-x} dy\,dx = \int_0^4 y\Big|_0^{x/2}dx + \int_4^6 y\Big|_0^{6-x}dx =$

$\int_0^4 x/2\,dx + \int_4^6 6-x\,dx = \frac{1}{4}x^2\Big|_0^4 + 6x-\frac{1}{2}x^2\Big|_4^6 = 4+36-18-24+8$

$=\boxed{6}$

i. $\int_0^{\pi/2}\int_0^3 re^{-r^2}\,dr\,d\theta = \int_0^{\pi/2} -\frac{1}{2}e^{-r^2}\Big|_0^3 d\theta = \int_0^{\pi/2} -\frac{1}{2}(e^{-9}-1)\,d\theta = \boxed{\frac{\pi}{4}(1-e^{-9})}$

2a. $\iint_R \sin x \sin y\,dA$



$\int_{-\pi}^{\pi}\int_0^{\pi/2} \sin x \sin y\,dy\,dx = \int_{-\pi}^{\pi} \sin x(-\cos y)\Big|_0^{\pi/2}dx = \int_{-\pi}^{\pi}\sin x\,dx = -\cos x\Big|_{-\pi}^{\pi}$

$-(-1)-[(-1)] = 1-1 = \boxed{0}$

b. $\iint_R xe^y\,dA$

$\int_0^4\int_0^{4-x} xe^y\,dy\,dx = \int_0^4 xe^y\Big|_0^{4-x}dx = \int_0^4 xe^{4-x}dx$

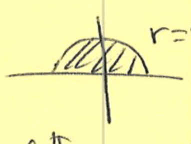
$-xe^{4-x} + \int_0^4 e^{4-x}dx = -xe^{4-x} - e^{4-x}\Big|_0^4 =$

$-4e^0 - e^0 + 0 + e^4 = \boxed{e^4-5}$

$u = x \quad dv = e^{4-x}dx$

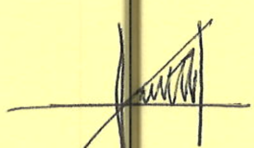
$du = dx \quad v = -e^{4-x}$

c. $\iint_R (x^2+y^2)\,dA$



$\int_0^{\pi}\int_0^2 r^2\cdot r\,dr\,d\theta = \int_0^{\pi}\frac{1}{4}r^4\Big|_0^2 d\theta = 4\int_0^{\pi}d\theta = \boxed{4\pi}$

3a. $z = xy,\ z = 0 \quad y = x,\ x = 1$



$\int_0^1\int_0^x xy\,dy\,dx = \int_0^1 \frac{1}{2}xy^2\Big|_0^x dx =$

$\frac{1}{2}\int_0^1 x^3\,dx = \frac{1}{8}x^4\Big|_0^1 = \boxed{\frac{1}{8}}$

3b. $z=\frac{1}{1+y^2}$, $x=0$, $x=2$, $y\ge 0$

$$\int_0^2\int_0^\infty \frac{1}{1+y^2}\,dy\,dx = \int_0^2 \arctan y\Big|_0^\infty dx = \int_0^2 \pi/2\, dx = \boxed{\pi}$$

c. $x^2+y^2+z^2=a^2$

$$z=\sqrt{a^2-x^2-y^2} \qquad z=-\sqrt{a^2-x^2-y^2}$$
$$=\sqrt{a^2-r^2} \qquad -\sqrt{a^2-r^2}$$

$$\int_0^{2\pi}\int_0^a 2\sqrt{a^2-r^2}\,r\,dr\,d\theta = 2\int_0^{2\pi}\int_{a^2}^{0} -\frac12 u^{1/2}\,du\,d\theta = 2\int_0^{2\pi}\frac12\cdot\frac23 u^{3/2}\Big|_0^{a^2} d\theta$$

$u=a^2-r^2$ $\quad r=a\to 0$

$du=-2r\,dr$ $\quad r=0\to a^2$

$-\frac12 du = r\,dr$

$$2\int_0^{2\pi}\frac13 (a^{3/2})^2\,d\theta = 2\left(\frac13 a^3\theta\right)\Big|_0^{2\pi} =$$

$$\boxed{\frac43 a^3\pi}$$

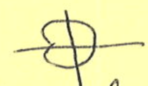
4a. $z=xy$, $x^2+y^2=1$, $y\ge 0$, $x\ge 0$, $z\ge 0$

$z=r^2\cos\theta\sin\theta$

$$\int_0^{\pi/2}\int_0^1 r^3\cos\theta\sin\theta\,dr\,d\theta = \int_0^{\pi/2}\frac14 r^4\Big|_0^1 \sin\theta\cos\theta\,d\theta = \frac14\int_0^{\pi/2}\frac12\sin 2\theta\,d\theta$$

$$-\frac18\cdot\frac12\cos 2\theta\Big|_0^{\pi/2} = -\frac1{16}(-1)+\frac1{16}(1) = \frac{2}{16} = \boxed{\frac18}$$

b. $f(r,\theta)=1+\sin(r)$ $\quad r=2+\cos\theta \quad 0\le\theta\le\pi/2$



$$\int_0^{\pi/2}\int_0^{2+\cos\theta}(1+\sin r)\,r\,dr\,d\theta =$$

$u=r$ $\quad dv=1+\sin r\,dr$

$du=dr$ $\quad v=r-\cos r$

$$\int_0^{\pi/2} r(r-\cos r) - \int_0^{\pi/2}\int_0^{2+\cos\theta} r-\cos r\,dr\,d\theta =$$

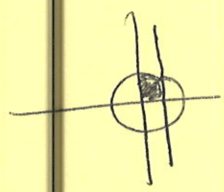
$$\int_0^{\pi/2} r^2-\cos r-\frac12 r^2+\sin r\Big|_0^{2+\cos\theta} d\theta = \int_0^{\pi/2}\frac12(2+\cos\theta)^2 - \cos(2+\cos\theta)+\sin(2+\cos\theta)\,d\theta$$

$$\int_0^{\pi/2}\frac12\left(4+4\cos\theta+\frac12\cos^2\theta\right)-\cos(2+\cos\theta)+\sin(2+\cos\theta)\,d\theta$$

do this numerically

≈ 7.3716

4c. $z=\sqrt{16-2r^2}$, inside $r=2$, $r=\sec\theta$



$$r=2=\sec\theta$$

$$\cos\theta=\frac{1}{2} \quad \theta=\pi/3$$

$$\int_0^{\pi/3}\int_0^{\sec\theta}\sqrt{16-2r^2}\,r\,dr\,d\theta+\int_{\pi/3}^{\pi/2}\int_0^{2}\sqrt{16-2r^2}\,r\,dr\,d\theta$$

$u=16-2r^2$ $u=16$ $r=0$

$du=-4r\,dr$ $u=8$ $r=2$

$-\frac{1}{4}du=r\,dr$

$-\frac{1}{4}\int u^{1/2}du=-\frac{1}{4}\cdot\frac{2}{3}u^{3/2}$

$$\int_0^{\pi/3}-\frac{1}{6}(16-2r^2)^{3/2}\Big|_0^{\sec\theta}d\theta+\int_{\pi/3}^{\pi/2}-\frac{1}{6}(16-2r^2)^{3/2}\Big|_0^{2}d\theta$$

$$\int_0^{\pi/3}-\frac{1}{6}(16-2\sec^2\theta)^{3/2}d\theta+\int_{\pi/3}^{\pi/2}-\frac{1}{6}(16-2(2)^2)^{3/2}+\frac{1}{6}(16-0)^{3/2}\,d\theta$$

$$\left(-\frac{\cancel{16}^{8}\sqrt{2}}{\cancel{6}3}+\frac{32}{3}\right)\theta\Big|_{\pi/3}^{\pi/2}$$

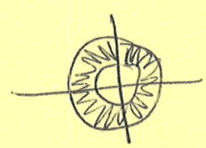
≈ -7.93509

(numerically)

$$\left(\frac{32}{3}-\frac{8\sqrt{2}}{3}\right)\frac{\pi}{6}$$

$\approx \sim 4.3246$

d. $z=\ln(x^2+y^2)$, $z=0$ $x^2+y^2\geq 1$, $x^2+y^2\leq 4$



$$\int_0^{2\pi}\int_1^2 2\ln r\cdot r\,dr\,d\theta$$

$u=\ln r$ $dv=r$

$du=\frac{1}{r}dr$ $v=\frac{1}{2}r^2$

$$\int_0^{2\pi}\frac{1}{2}r^2\ln r-\int_1^2\frac{1}{2}r\,dr\,d\theta=$$

$$\int_0^{2\pi}\frac{1}{2}r^2\ln r-\frac{1}{4}r^2\Big|_1^2 d\theta=\int_0^{2\pi}\frac{1}{2}(2)^2\ln 2-\frac{1}{4}(2)^2-\frac{1}{2}(1)^2\ln(1)+\frac{1}{4}(1)^2\,d\theta$$

$$\int_0^{2\pi}\left(2\ln 2-\frac{3}{4}\right)d\theta=\boxed{2\pi\left(2\ln 2-\frac{3}{4}\right)}$$

Cube 2x2x2 centered at origin

5a. $$\int_{-1}^1\int_{-1}^1\int_{-1}^1 x^2y^2z^2\,dx\,dy\,dz=\int_{-1}^1\int_{-1}^1\frac{1}{3}x^3y^2z^2\Big|_{-1}^1 dy\,dz=\frac{2}{3}\int_{-1}^1\int_{-1}^1 y^2z^2\,dy\,dz=$$

$$\frac{2}{3}\int_{-1}^1\frac{1}{3}y^3z^2\Big|_{-1}^1 dz=\frac{4}{9}\int_{-1}^1 z^2\,dz=\frac{4}{9}\cdot\frac{1}{3}z^3\Big|_{-1}^1=\boxed{\frac{8}{27}}$$

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5b. $\int_0^{\pi/2}\int_0^{y/2}\int_0^{1/y} \sin y\, dz\,dx\,dy = \int_0^{\pi/2}\int_0^{y/2} z\sin y\Big|_0^{1/y} dx\,dy = \int_0^{\pi/2}\int_0^{y/2} \frac{1}{y}\sin y\, dx\,dy$

$= \int_0^{\pi/2} \frac{x}{y}\sin y\Big|_0^{y/2} dy = \int_0^{\pi/2} \frac{1}{2}\sin y\, dy = -\frac{1}{2}\cos y\Big|_0^{\pi/2} = 0 + \frac{1}{2}(1) = \boxed{\frac{1}{2}}$

c. $\int_0^2\int_{2x}^4\int_0^{\sqrt{y^2-4x^2}} dz\,dy\,dx = \int_0^2\int_{2x}^4 z\Big|_0^{\sqrt{y^2-4x^2}} dy\,dx = \int_0^2\int_{2x}^4 \sqrt{y^2-4x^2}\,dy\,dx$

elliptic cone wedge

≈ 8.37758

d. $\int_0^{\pi/4}\int_0^2\int_0^{2-r} dz\,dr\,d\theta = \int_0^{\pi/4}\int_0^2 z\Big|_0^{2-r} dr\,d\theta = \int_0^{\pi/4}\int_0^2 2-r\,dr\,d\theta =$

$\int_0^{\pi/4} 2r - \frac{1}{2}r^2\Big|_0^2 d\theta = \int_0^{\pi/4} 4-2\,d\theta = \int_0^{\pi/4} 2\,d\theta = 2\theta\Big|_0^{\pi/4} = \boxed{\pi/2}$

e. $\int_0^{\pi/2}\int_0^{\pi}\int_0^{\sin\theta} 2\cos\varphi\,\rho^2\,d\rho\,d\theta\,d\varphi = \int_0^{\pi/2}\int_0^{\pi} \frac{2}{3}\rho^3\Big|_0^{\sin\theta} d\theta\,d\varphi =$

$\int_0^{\pi/2}\int_0^{\pi} \frac{2}{3}\sin^3\theta\,d\theta\,d\varphi = \frac{2}{3}\int_0^{\pi/2}\int_0^{\pi}(1-\cos^2\theta)\sin\theta\,d\theta\,d\varphi =$

$u = \cos\theta$
$du = -\sin\theta$

$-\frac{2}{3}\int_0^{\pi/2}\left(\theta - \frac{1}{3}\cos^3\theta\right)\Big|_0^{\pi} d\varphi = -\frac{2}{3}\int_0^{\pi/2} \pi - \frac{1}{3}(-1)^3 - 0 + \frac{1}{3}(1)^3\,d\varphi$

$-\frac{2}{3}\int_0^{\pi/2}\left(\pi + \frac{2}{3}\right)d\varphi = -\frac{2}{3}\left(\pi+\frac{2}{3}\right)\varphi\Big|_0^{\pi/2} = -\frac{\cancel{2}^{1}}{3}\frac{\pi^2}{\cancel{6}_{3}} - \frac{\cancel{4}^{2}}{9}\cdot\frac{\pi}{\cancel{2}} = \boxed{-\frac{\pi^2}{9} - \frac{2\pi}{9}}$

f. $\int_0^2\int_0^{\sqrt{4-x^2}}\int_0^{\sqrt{4-x^2-y^2}} \sqrt{x^2+y^2}\,dz\,dy\,dx$

$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^2 \rho\sin\varphi\cdot\rho^2\sin^2\varphi\,d\rho\,d\varphi\,d\theta = \int_0^{\pi/2}\int_0^{\pi/2}\int_0^2 \rho^3\sin^3\varphi\,d\rho\,d\varphi\,d\theta =$

$\int_0^{\pi/2}\int_0^{\pi/2} \frac{1}{4}\rho^4\Big|_0^2 (1-\cos^2\varphi)\sin\varphi\,d\varphi\,d\theta = -4\int_0^{\pi/2}\int_0^{\pi/2}(\cos^2\varphi - 1)(\sin\varphi)\,d\varphi\,d\theta$

$-4\int_0^{\pi/2} \varphi - \frac{1}{3}\cos^3\varphi\Big|_0^{\pi/2} d\theta = -4\int_0^{\pi/2} \frac{\pi}{2} - 0 - 0 + \frac{1}{3}(1)\,d\theta = \boxed{-4\left(\frac{\pi}{2}+\frac{1}{3}\right)\frac{\pi}{2}}$

5g. $\int_1^4\int_1^{e^2}\int_0^{1/xz} \ln z\,dy\,dz\,dx = \int_1^4\int_1^{e^2} y\ln z\Big|_0^{1/xz} dz\,dx =$

$\int_1^4\int_1^{e^2} \frac{1}{x}\cdot\frac{\ln z}{z}\, dz\,dx = \int_1^4 \frac{1}{x}\frac{(\ln z)^2}{2}\Big|_1^{e^2} dx = \int_1^4 \frac{2}{x}\,dx = 2\ln x\Big|_1^4 =$

$\boxed{2\ln 4}$

h. $\int_0^2\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_0^{x^2} y\,dz\,dy\,dx = \int_{-\pi/2}^{\pi/2}\int_0^2\int_0^{r^2\cos^2\theta} r^2\sin\theta\,dz\,dr\,d\theta =$

$\int_{-\pi/2}^{\pi/2}\int_0^2 z r^2\sin\theta\Big|_0^{r^2\cos^2\theta} dr\,d\theta = \int_{-\pi/2}^{\pi/2}\int_0^2 r^4\sin\theta\cos^2\theta\,dr\,d\theta$

$\int_{-\pi/2}^{\pi/2}\frac{1}{5}r^5\Big|_0^2 \sin\theta\cos^2\theta\,d\theta = \int_{-\pi/2}^{\pi/2}\frac{32}{5}\sin\theta\cos^2\theta\,d\theta = -\frac{32}{5}\cdot\frac{1}{3}\cos^3\theta\Big|_{-\pi/2}^{\pi/2}$

$=\boxed{0}$

i. $\int_0^3\int_0^x\int_0^{9-x^2} dz\,dy\,dx = \int_0^3\int_0^x z\Big|_0^{9-x^2}dy\,dx = \int_0^3\int_0^x 9-x^2\,dy\,dx =$

$\int_0^3 9y-x^2y\Big|_0^x dx = \int_0^3 9x-x^3\,dx = \frac{9}{2}x^2-\frac{1}{4}x^4\Big|_0^3 = \frac{81}{2}-\frac{81}{4} = \boxed{\frac{81}{4}}$

j. $\int_0^{\pi/4}\int_0^{\pi/4}\int_0^{\cos\theta}\rho^2\sin\varphi\cos\varphi\,d\rho\,d\theta\,d\varphi = \int_0^{\pi/4}\int_0^{\pi/4}\frac{1}{3}\rho^3\sin\varphi\cos\varphi\Big|_0^{\cos\theta}d\theta\,d\varphi =$

$\frac{1}{3}\int_0^{\pi/4}\int_0^{\pi/4}\cos^3\theta\sin\varphi\cos\varphi\,d\theta\,d\varphi = \frac{1}{3}\int_0^{\pi/4}\int_0^{\pi/4}(1-\sin^2\theta)\cos\theta\sin\varphi\cos\varphi\,d\theta\,d\varphi$

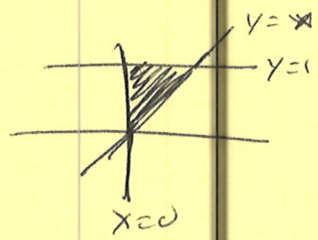
$=\frac{1}{3}\int_0^{\pi/4}(\theta-\frac{1}{3}\sin^3\theta\Big|_0^{\pi/4}\sin\varphi\cos\varphi\,d\varphi = \frac{1}{3}\int_0^{\pi/4}(\frac{\pi}{4}-\frac{1}{3}\cdot\frac{1}{2\sqrt2})\sin\varphi\cos\varphi\,d\varphi =$

$\frac{1}{3}(\frac{\pi}{4}-\frac{1}{6\sqrt2})\frac{1}{2}\sin^2\varphi\Big|_0^{\pi/4} = \frac{1}{6}\cdot\frac{1}{2}(\frac{\pi}{4}-\frac{1}{6\sqrt2}) = \boxed{\frac{\pi}{48}-\frac{1}{72\sqrt2}}$

k. $\int_0^{2\pi}\int_0^{\sqrt3}\int_0^{3-r^2} r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^{\sqrt3} rz\Big|_0^{3-r^2}dr\,d\theta = \int_0^{2\pi}\int_0^{\sqrt3}3r-r^3\,dr\,d\theta$

$\int_0^{2\pi}\frac{3}{2}r^2-\frac{1}{4}r^4\Big|_0^{\sqrt3}d\theta = \int_0^{2\pi}\frac{9}{2}-\frac{9}{4}\,d\theta = \int_0^{2\pi}\frac{9}{4}\,d\theta = \frac{9}{4}\theta\Big|_0^{2\pi} = \boxed{\frac{9\pi}{2}}$

6a. $\int_0^1\int_x^1 x\sqrt{1+2y^3}\,dy\,dx$



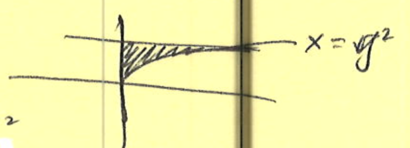
$\int_0^1\int_0^y x\sqrt{1+2y^3}\,dx\,dy$

$\int_0^1\int_0^y \frac{1}{2}x^2\Big|_0^y\sqrt{1+2y^3}\,dy = \int_0^1 \frac{1}{2}y^2\sqrt{1+2y^3}\,dy$

$1+2y^3=u$ $\genfrac{}{}{0pt}{}{3}{1}$
$6y^2 = du$
$y^2dy = \frac{1}{6}du$

$\frac{1}{12}\int_1^3 u^{1/2}\,du = \frac{1}{12}\cdot\frac{2}{3}u^{3/2}\Big|_1^3 = \boxed{\frac{1}{18}\left(3^{3/2}-1\right)}$

b. $\int_0^9\int_{\sqrt{x}}^3 \frac{4}{5+y^3}\,dy\,dx$

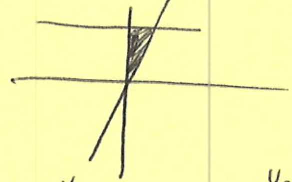


$\int_0^3\int_0^{y^2}\frac{4}{5+y^3}\,dx\,dy = \int_0^3 \frac{4x}{5+y^3}\Big|_0^{y^2}dy = \int_0^3\frac{4y^2}{5+y^3}\,dy = \frac{4}{3}\ln|5+y^3|\Big|_0^3$

$\boxed{\frac{4}{3}\left(\ln(32)-\ln 5\right)}$

$y=3x \rightarrow x=\frac{1}{3}y$

c. $\int_0^1\int_{3x}^3 6e^{y^2}\,dy\,dx$



$\int_0^{1/3}\int_0^{1/3y} 6e^{y^2}\,dx\,dy = \int_0^{1/3} 6e^{y^2}x\Big|_0^{1/3y}dy = \int_0^{1/3} 2e^{y^2}\cdot y\,dy = e^{y^2}\Big|_0^{1/3} =$

$e^{1/9}-1$

7a. $z=9-x^2$ $\quad y=2x$

$\int_0^2\int_0^{2x}\int_0^{9-x^2}dz\,dy\,dx = \int_0^2\int_0^{2x}9-x^2\,dy\,dx = \int_0^2 18x-\frac{1}{3}(2x)^3\,dx = 9x^2-\frac{8}{3}\cdot\frac{1}{4}x^4\Big|_0^2$

$= 36-\frac{32}{3} = \boxed{\frac{76}{3}}$

b. $\int_0^{\pi/2}\int_0^y\int_1^{1/y}\sin y\,dz\,dx\,dy \Rightarrow$ see 16.

8.a. $\rho=4,\ \varphi=\pi/4$

$2\int_{\pi/4}^{\pi/2}\int_0^{2\pi}\int_0^4\rho^2\sin\varphi\,d\rho\,d\theta\,d\varphi = \frac{2}{3}\int_{\pi/4}^{\pi/2}\int_0^{2\pi}\rho^3\Big|_0^4\sin\varphi\,d\theta\,d\varphi = \frac{128}{3}\int_{\pi/4}^{\pi/2}2\pi\sin\varphi\,d\varphi$

$\frac{256\pi}{3}(-\cos\varphi)\Big|_{\pi/4}^{\pi/2} = \frac{256\pi}{3}\left(0+\frac{1}{\sqrt{2}}\right) = \boxed{\frac{256\pi}{3\sqrt{2}}}$

8b. $\displaystyle\int_0^{\cos^{-1}\left(\frac{8}{\sqrt{80}}\right)}\int_0^{2\pi}\int_{2\cot\varphi\csc\varphi}^{\sqrt{80}} \rho^2\sin\varphi\, d\rho\, d\theta\, d\varphi$

$\rho\cos\varphi = \frac{1}{2}\rho^2\sin^2\varphi$

$2\cot\varphi\csc\varphi = \rho$

$\displaystyle\int_0^{\cos^{-1}\left(\frac{8}{\sqrt{80}}\right)}\int_0^{2\pi} \frac{1}{3}\rho^3\Big|_{2\cot\varphi\csc\varphi}^{\sqrt{80}} \sin\varphi\, d\theta\, d\varphi$

$x^2+y^2 = 80 - z^2$

$z = \frac{1}{2}(80 - z^2)$

$\displaystyle\frac{1}{3}\int_0^{\cos^{-1}\left(\frac{8}{\sqrt{80}}\right)}\int_0^{2\pi} 80\sqrt{80} - 8\cot^3\varphi\csc^3\varphi\sin\varphi\, d\theta\, d\varphi$

$z^2 + 2z - 80 = 0$

$(z+10)(z-8) = 0 \quad z = -10,\ z = 8$

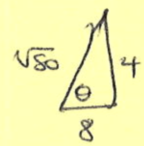
$\displaystyle\frac{1}{3}\int_0^{\cos^{-1}\left(\frac{8}{\sqrt{80}}\right)}\int_0^{2\pi} 80\sqrt{80} - 8\cot^3\varphi\csc^2\varphi\, d\theta\, d\varphi$

$8 = \sqrt{80}\cos\varphi$

$\frac{8}{\sqrt{80}} = \cos\varphi$

$\displaystyle\frac{2\pi}{3}\int_0^{\cos^{-1}\left(\frac{8}{\sqrt{80}}\right)} 80\sqrt{80} - 8\cot^3\varphi\csc^2\varphi\, d\varphi$

$\displaystyle\frac{2\pi}{3}\left[80\sqrt{80}\cos^{-1}\left(\frac{8}{\sqrt{80}}\right) + \frac{8}{4}\cot^4\varphi\Big|_0^{\cos^{-1}\left(\frac{8}{\sqrt{80}}\right)}\right] =$



$\displaystyle\frac{2\pi}{3}\left[80\sqrt{80}\cos^{-1}\left(\frac{8}{\sqrt{80}}\right) + 2\left(\frac{8}{4}\right)^4\right] = \frac{2\pi}{3}\left[80\sqrt{80}\cos^{-1}\left(\frac{8}{\sqrt{80}}\right) + 32\right]$

8c.

$z = \frac{h}{a}r \qquad \rho\cos\varphi = \frac{h}{a}\rho\sin\varphi \qquad z = h$

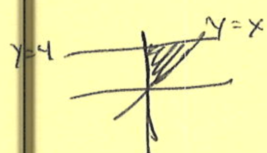
$\frac{a}{h} = \tan\varphi \qquad \rho\cos\varphi = h$

$\displaystyle\int_0^{\tan^{-1}\left(\frac{a}{h}\right)}\int_0^{2\pi}\int_0^{h\sec\varphi} \rho^2\sin\varphi\, d\rho\, d\theta\, d\varphi = \int_0^{\tan^{-1}\left(\frac{a}{h}\right)}\int_0^{2\pi} \frac{1}{3}\rho^3\Big|_0^{h\sec\varphi} \sin\varphi\, d\theta\, d\varphi =$

$\displaystyle\int_0^{\tan^{-1}\left(\frac{a}{h}\right)}\int_0^{2\pi} \frac{h^3}{3}\sec^3\varphi\sin\varphi\, d\theta\, d\varphi = \int_0^{\tan^{-1}\left(\frac{a}{h}\right)} \frac{2}{3}\pi h^3\sec^2\varphi\tan\varphi\, d\varphi =$

$\displaystyle\frac{2}{3}\pi h^3\,\frac{1}{2}\tan^2\varphi\Big|_0^{\tan^{-1}\left(\frac{a}{h}\right)} = \frac{2}{3}\pi h^3\cdot\frac{1}{2}\left(\frac{a}{h}\right)^2 = \boxed{\frac{1}{3}\pi a^2 h}$

9a. $\displaystyle\int_0^4\int_x^4 e^{-y^2}\, dy\, dx = \int_0^4\int_0^y e^{-y^2}\, dx\, dy =$



$\displaystyle\int_0^4 ye^{-y^2}\, dy = -\frac{1}{2}e^{-y^2}\Big|_0^4 = \boxed{-\frac{1}{2}e^{-16} + \frac{1}{2}}$

9b. $\int_0^1 \int_{y^2}^1 \sqrt{x}\sin x \, dx\, dy$

$\int_0^1 \int_0^{\sqrt{x}} \sqrt{x}\sin x \, dy\, dx = \int_0^1 x\sin x \, dx$

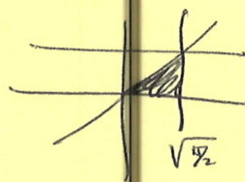
$u = x \quad dv = \sin x\, dx$

$du = dx \quad v = -\cos x$

$-x\cos x + \int \cos x\, dx = -x\cos x + \sin x \Big|_0^1$

$-1\cos 1 + \sin 1 + 0 + 0 = \boxed{\sin 1 - \cos 1}$

c. $\int_0^{\sqrt{\pi/2}} \int_y^{\sqrt{\pi/2}} \sin x^2 \, dx\, dy$



$\int_0^{\sqrt{\pi/2}} \int_0^{x} \sin x^2 \, dy\, dx = \int_0^{\sqrt{\pi/2}} x\sin x^2\, dx$

$= -\frac{1}{2}\cos(x^2) \Big|_0^{\sqrt{\pi/2}} = -\frac{1}{2}(0) + \frac{1}{2}(1) = \boxed{\frac{1}{2}}$

10.a. $\int_0^1 \int_{-\sqrt{x-x^2}}^{\sqrt{x-x^2}} \underbrace{(x^2+y^2)^{3/2}}_{\to r^3} dy\, dx$

$y = \sqrt{x-x^2}$

$y^2 = x - x^2$

$x^2 + y^2 = x$

$r^2 = r\cos\theta$

$r = \cos\theta$

$\int_0^{\pi} \int_0^{\cos\theta} r^4\, dr\, d\theta = \int_0^{\pi} \frac{1}{5} r^5 \Big|_0^{\cos\theta} d\theta$

$\frac{1}{5}\int_0^{\pi} \cos^5\theta\, d\theta = \frac{1}{5}\int_0^{\pi} \cos\theta(1-\sin^2\theta)^2\, d\theta$

$\frac{1}{5}\int_0^{\pi} \cos\theta(1 - 2\sin^2\theta + \sin^4\theta)\, d\theta = \frac{1}{5}\left[\theta - \frac{2}{3}\sin^3\theta + \frac{1}{5}\sin^5\theta\right|_0^{\pi}$

$\frac{1}{5}[\pi - 0 + 0 - 0 + 0 - 0] = \boxed{\frac{\pi}{5}}$

b. $\int_0^6 \int_0^{\sqrt{6y-y^2}} x^2\, dy\, dx$

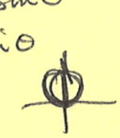
$x = \sqrt{6y - y^2}$

$x^2 + y^2 = 6y$

$r^{\cancel{2}} = 6\cancel{r}\sin\theta$

$r = 6\sin\theta$

$\int_0^{\pi} \int_0^{6\sin\theta} r^3\cos^2\theta\, dr\, d\theta = \int_0^{\pi} \frac{1}{4} r^4 \Big|_0^{6\sin\theta} \cos^2\theta\, d\theta$



$324\int_0^{\pi} \sin^4\theta\cos^2\theta\, d\theta \quad \approx 63.62$

c. $\int_0^2 \int_y^{\sqrt{8-y^2}} \sin\sqrt{x^2+y^2}\, dy\, dx = \int_{\pi/4}^{\pi/2} \int_0^{\sqrt{8}} r\sin r\, dr\, d\theta$

$u = r \quad dv = \sin r\, dr$

$du = dr \quad v = -\cos r$

$\int_{\pi/4}^{\pi/2} -r\cos r + \int \cos r\, dr\; d\theta = \int_{\pi/4}^{\pi/2} -r\cos r + \sin r \Big|_0^{\sqrt{8}} d\theta$

$(-\sqrt{8}\cos\sqrt{8} + \sin\sqrt{8})\int_{\pi/4}^{\pi/2} d\theta = \boxed{(\sin\sqrt{8} - \sqrt{8}\cos\sqrt{8})\frac{\pi}{4}}$

11a. $\int_0^2\int_0^{\sqrt{4-x^2}}\int_0^{\sqrt{4-x^2-y^2}} \sqrt{x^2+y^2}\,dz\,dy\,dx$

$\int_0^{\pi/2}\int_0^2\int_0^{\sqrt{4-r^2}} r\cdot r\,dz\,dr\,d\theta$ See 1f.

b. $\int_0^{\pi/2}\int_0^{\pi/2}\int_0^2 \rho\sin\varphi\cdot\rho^2\sin\varphi\,d\rho\,d\theta\,d\varphi$ See 1f.

c. $\int_0^5\int_0^{\sqrt{25-x^2}}\int_0^{\sqrt{25-x^2-y^2}} \frac{1}{1+x^2+y^2+z^2}\,dz\,dy\,dx$

$= \int_0^{\pi/2}\int_0^{\pi/2}\int_0^5 \frac{1}{1+\rho^2}\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$

$(\tan^{-1}(1/5) - \pi/2 + 5)\int_0^{\pi/2}\sin\varphi\,d\varphi\cdot\pi/2 = \frac{\pi}{2}(\arctan(\frac{1}{5}) - \frac{\pi}{2} + 5)$

d. $\int_{-2}^2\int_0^{\sqrt{4-x^2}}\int_{\sqrt{3x^2+3y^2}}^{\sqrt{16-x^2-y^2}} \cos(x^2+y^2)\,dz\,dy\,dx$

$16-x^2-y^2=3x^2+3y^2$
$16=4x^2+y^2$
$r=2$

$\int_0^\pi\int_0^2\int_{\sqrt{3}r}^{\sqrt{16-r^2}} \cos r^2\cdot r\,dz\,dr\,d\theta$

$\int_0^\pi\int_0^2 r\cos r^2(\sqrt{16-r^2}-\sqrt{3}r)\,dr\,d\theta$

$\pi\int_0^2 r\cos r^2(\sqrt{16-r^2}-\sqrt{3}r)\,dr \approx \boxed{.804\pi}$

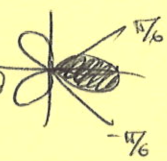
e. $\int_{-2}^2\int_0^{\sqrt{4-x^2}}\int_{\sqrt{3x^2+3y^2}}^{\sqrt{16-x^2-y^2}} \cos(\sqrt{x^2+y^2+z^2})\,dz\,dy\,dx$

$z=\sqrt{3}r$
$\rho\cos\varphi=\sqrt{3}\rho\sin\varphi$
$\frac{1}{\sqrt{3}}=\tan\varphi$

$\int_0^{\pi/6}\int_0^\pi\int_0^4 \cos\rho\cdot\rho^2\sin\varphi\,d\rho\,d\theta\,d\varphi$

$\pi(8\cos 4+14\sin 4)\int_0^{\pi/6}\sin\varphi\,d\varphi = \pi(8\cos 4+14\sin 4)(1-\frac{\sqrt{3}}{2})$

12. $\int_{-\pi/6}^{\pi/6}\int_0^{\cos 3\theta} r\,dr\,d\theta = \int_{-\pi/6}^{\pi/6}\frac{1}{2}(\cos^2 3\theta)\,d\theta = \int_{-\pi/6}^{\pi/6}\frac{1}{4}(1+\cos 6\theta)\,d\theta$



$\frac{1}{4}\left[\theta+\frac{1}{6}\sin 6\theta\right]_{-\pi/6}^{\pi/6} = \boxed{\frac{\pi}{12}}$