MTH 277, Quiz #10, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Find the unit vector and unit normal vector for $\vec{r}(t) = 4\cos 3t\,\hat{\imath} + 4\sin 3t\,\hat{\jmath} + t\hat{k}$. Use that information to find an equation of the tangent line and the normal line at $t = \frac{\pi}{3}$.

$r'(t) = -12\sin 3t\,\hat{\imath} + 12\cos 3t\,\hat{\jmath} + \hat{k}$

$\|r'(t)\| = \sqrt{144\sin^2 3t + 144\cos^2 3t + 1} = \sqrt{145}$

$r(\pi/3) = 4\cos\pi\,\hat{\imath} + 4\sin\pi\,\hat{\jmath} + \pi/3\,\hat{k} = \langle -4, 0, \pi/3 \rangle$

$T(t) = \dfrac{r'(t)}{\|r'(t)\|} = \dfrac{-12}{\sqrt{145}}\sin 3t\,\hat{\imath} + \dfrac{12}{\sqrt{145}}\cos 3t\,\hat{\jmath} + \dfrac{1}{\sqrt{145}}\hat{k}$

$T(\pi/3) = \dfrac{-12}{\sqrt{145}}\sin\pi\,\hat{\imath} + \dfrac{12}{\sqrt{145}}\cos(\pi)\,\hat{\jmath} + \dfrac{1}{\sqrt{145}}\hat{k} =$

$\left\langle 0, \dfrac{-12}{\sqrt{145}}, \dfrac{1}{\sqrt{145}} \right\rangle$ unit tangent

$N(t) = \dfrac{T'(t)}{\|T'(t)\|}$

$T'(t) = \dfrac{-36}{\sqrt{145}}\cos 3t\,\hat{\imath} - \dfrac{36}{\sqrt{145}}\sin 3t\,\hat{\jmath} + 0\hat{k}$

$\|T'(t)\| = \dfrac{36}{\sqrt{145}}$

$N(t) = -\cos 3t\,\hat{\imath} - \sin 3t\,\hat{\jmath}$

$N(\pi/3) = -\cos\pi\,\hat{\imath} - \sin\pi\,\hat{\jmath} = \langle 1, 0, 0 \rangle$ unit normal

tangent line $\tau(t) = (0t - 4)\hat{\imath} + \left(-\dfrac{12}{\sqrt{145}}t + 0\right)\hat{\jmath} + \left(\dfrac{1}{\sqrt{145}}t + \pi/3\right)\hat{k}$

$= -4\hat{\imath} + (-12t)\hat{\jmath} + (t + \pi/3)\hat{k}$

normal line $n(t) = (1t - 4)\hat{\imath} + (0t + 0)\hat{\jmath} + (0t + \pi/3)\hat{k} =$

$(t-4)\hat{\imath} + 0\hat{\jmath} + \pi/3\,\hat{k}$