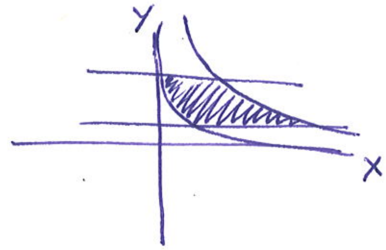
MTH 277, Quiz #20, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Use a change of variables to evaluate $\int_R \int y \sin xy \, dA$ for the region bounded by $xy = 1, xy = 8, y = 1, y = 2$.

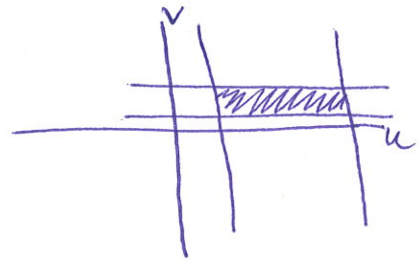


$u = xy \quad [1,8]$

$v = y \quad [1,2]$

$u = xv$

$\frac{u}{v} = x$



$$J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{1}{v} & \frac{-u}{v^2} \\ 0 & 1 \end{vmatrix} = \frac{1}{v}$$

$$\int_1^2 \int_1^8 \cancel{y} \sin u \cdot \frac{1}{\cancel{v}} \, du\,dv = \int_1^2 \int_1^8 \sin u \, du\,dv = \int_1^2 \cos u \Big|_1^8 dv =$$

$$(2-1)(\cos 8 - \cos 1) \approx -.6858$$

2. Find the Jacobian for $x = uv - 2u, y = uv$.

$$J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} v-2 & u \\ v & u \end{vmatrix} = (v-2)u - vu = \cancel{uv} - 2u - \cancel{vu} = -2u$$

$$\rightarrow |J| = 2u$$

3. Find the position vector for $\vec{a}(t) = 2t\hat{\imath} - \frac{1}{t^2}\hat{k}, \vec{v}(1) = \hat{\imath} + \hat{\jmath}, \vec{r}(1) = 2\hat{\jmath} - \hat{k}$.

$$v(t) = \int \left(2t\hat{\imath} - \frac{1}{t^2}\hat{k}\right) dt = (t^2 + C_1)\hat{\imath} + C_2\hat{\jmath} + \left(\frac{1}{t} + C_3\right)\hat{k}$$

$v(1) = 1 \cdot \hat{\imath} + 1\hat{\jmath} + 0\hat{k}$

$C_1 = 0 \quad C_2 = 1 \quad C_3 = -1$

$$r(t) = \int (t^2)\hat{\imath} + 1\hat{\jmath} + \left(\frac{1}{t} - 1\right)\hat{k} \, dt = \left(\frac{1}{3}t^3 + C_1\right)\hat{\imath} + (t + C_2)\hat{\jmath} + (\ln t - t + C_3)\hat{k}$$

$r(1) = 0\hat{\imath} + 2\hat{\jmath} - \hat{k}$

$\frac{1}{3} + C_1 = 0 \quad C_1 = -\frac{1}{3}$

$1 + C_2 = 2 \quad C_2 = 1$

$0 - 1 + C_3 = -1 \quad C_3 = 0$

$$\boxed{r(t) = \left(\frac{1}{3}t^3 - \frac{1}{3}\right)\hat{\imath} + (t+1)\hat{\jmath} + (\ln t - t)\hat{k}}$$