MTH 277, Quiz #9, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Find the volume below the surface $f(x,y) = 2xy$ on the rectangle on $[0,5] \times [0,3]$.

$$\int_0^5 \int_0^3 2xy\,dy\,dx = \int_0^5 xy^2\,dx\Big|_0^3 = \int_0^5 9x\,dx =$$

$$\frac{9}{2}x^2\Big|_0^5 = \frac{9 \cdot 25}{2} = \boxed{\frac{225}{2}}$$

2. Use a double integral to find the volume bounded by $z = \sqrt{x^2+y^2}, z = 0, x^2+y^2 = 25$.

$z = r$

$$\int_0^{2\pi}\int_0^5 \underbrace{r \cdot r}_{r^2}\,dr\,d\theta = \int_0^{2\pi} \frac{1}{3}r^3\Big|_0^5 d\theta =$$

$$\int_0^{2\pi} \frac{125}{3}\,d\theta = \frac{125}{3}\theta\Big|_0^{2\pi} = \boxed{\frac{250\pi}{3}}$$

3. Integrate $\int_1^4 \int_1^{e^2} \int_0^{1/xz} \ln z\,dy\,dz\,dx$.

$$\int_1^4 \int_1^{e^2} y \ln z\Big|_0^{1/xz} dz\,dx = \int_1^4 \frac{1}{x}\int_1^{e^2} \frac{\ln z}{z}\,dz\,dx$$

$u = \ln z$
$du = \frac{1}{z}dz$ $\int \frac{\ln z}{z}dz = \int u\,du$

$$\int_1^4 \frac{1}{x} \cdot \frac{1}{2}(\ln z)^2\Big|_1^{e^2} dx = \int_1^4 \frac{1}{2x}\left[(\ln e^2)^2 - \ln(0)^2\right] dx =$$

$$\frac{1}{2}\int_1^4 \frac{1}{x}(4)\,dx = 2\ln x\Big|_1^4 = \boxed{2\ln 4}$$