MTH 291, Quiz #12, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Consider the solutions to a second order differential equation in the table below, and the given forcing function. What is the appropriate Ansatz for the method of undetermined coefficients?

	$y_1(x)$	$y_2(x)$	$f(x)$	$Y(x)$
a.	e^{-x}	e^{-6x}	$\frac{1}{2}\sin(x)$	$A\sin x + B\cos x$
b.	$\sin(2x)$	$\cos(2x)$	$2e^{-x} + \cos 2x$	$Ae^{-x} + Bx\sin 2x + Cx\cos 2x$
c.	$e^{-x}\cos 3x$	$e^{-x}\sin 3x$	$9\sin 3x$	$A\sin 3x + B\cos 3x$
d.	e^{-2x}	e^{-x}	$x^3 + 1$	$Ax^3 + Bx^2 + Cx + D$

2. Consider the differential equation $y'' + 7y' + 12y = 3\sin 2t$. Find the particular solution to the non-homogenous differential equation. Final all undetermined coefficients. Which part of the solution is transient and which is steady state?

$r^2 + 7r + 12 = 0$

$(r+3)(r+4) = 0$

$r = -3, r = -4$

$y_g = c_1 e^{-3t} + c_2 e^{-4t}$

$Y_p = A\sin 2t + B\cos 2t$

$Y_p' = 2A\cos 2t - 2B\sin 2t$

$Y_p'' = -4A\sin 2t - 4B\cos 2t$

$-4A\sin 2t - 4B\cos 2t + 7(2A\cos 2t - 2B\sin 2t) + 12(A\sin 2t + B\cos 2t) = 3\sin 2t$

$-4A\sin 2t - 4B\cos 2t + 14A\cos 2t - 14B\sin 2t + 12A\sin 2t + 12B\cos 2t = 3\sin 2t$

$\sin 2t\,(-4A - 14B + 12A) = 3\sin 2t$ $\quad 8A - 14B = 3$ $\quad A = 6/65$

$\cos 2t\,(-4B + 14A + 12B) = 0$ $\quad 14A + 8B = 0$ $\quad B = -21/130$

$y(t) = \underbrace{c_1 e^{-3t} + c_2 e^{-4t}}_{\text{transient}} + \underbrace{\frac{6}{65}\sin 2t - \frac{21}{130}\cos 2t}_{\text{steady state}}$