Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. For what values of r does $y = e^{rx}$ satisfy the differential equation $2y'' + y' - y = 0$?

$$y' = re^{rx}$$
$$y'' = r^2 e^{rx}$$

$$2r^2 e^{rx} + re^{rx} - e^{rx} = 0$$

$$e^{rx}(2r^2 + r - 1) = 0$$

→ never zero

$$(2r - 1)(r + 1) = 0$$

$$r = \tfrac{1}{2}, \ r = -1$$

2. Rewrite the second-order equation above as a system, and use the system to describe the behavior of the system using the phase plane.

$$y = x_1$$
$$y' = x_1' = x_2$$
$$x_2' = y''$$

$$2x_2' + x_2 - x_1 = 0$$

$$x_2' = \frac{x_1 - x_2}{2}$$

$$\boxed{\begin{aligned} x_1' &= x_2 \\ x_2' &= \tfrac{1}{2}x_1 - \tfrac{1}{2}x_2 \end{aligned}}$$