MTH 291, Quiz #9, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. A mass weighing 16 pounds stretches a spring $\frac{8}{3}$ of a foot. The mass is initially released from a point 2 feet below the equilibrium position, and the subsequent motion takes place in a medium that offers a damping force equivalent to $\frac{1}{2}$ the instantaneous velocity. Write the equation of the spring-mass system (either as a linear system or a second-order equation.)

$F = 16 = k\left(\frac{8}{3}\right)$ $\quad k = 6$

$F = mg$ $\quad 16 = m(32)$ $\quad m = \frac{1}{2}$

$\gamma = \frac{1}{2}$

$$\frac{1}{2}y'' + \frac{1}{2}y' + 6y = 0$$
$$y(0) = -2,\ y'(0) = 0$$

$$y'' + y' + 12y = 0$$

$$x_1' = x_2$$
$$x_2' = -12x_1 - x_2$$
$$x_1(0) = -2,\ x_2(0) = 0$$

Find the particular solution for the given initial conditions.

$$r^2 + r + 12 = 0$$

$$r = \frac{-1 \pm \sqrt{1 - 48}}{2} = \frac{-1 \pm \sqrt{47}\,i}{2}$$

$$y(t) = c_1 e^{-\frac{t}{2}} \cos\left(\tfrac{\sqrt{47}}{2}t\right) + c_2 e^{-\frac{t}{2}} \sin\left(\tfrac{\sqrt{47}}{2}t\right)$$

$$y(0) = c_1 e^0 \cos(0) + c_2 e^0 \sin(0) = -2$$

$$c_1 = -2$$

$$y(t) = -2e^{-\frac{t}{2}} \cos\left(\tfrac{\sqrt{47}}{2}t\right) + c_2 e^{-\frac{t}{2}} \sin\left(\tfrac{\sqrt{47}}{2}t\right)$$

$$y'(t) = \frac{2}{2}e^{-\frac{t}{2}} \cos\left(\tfrac{\sqrt{47}}{2}t\right) - \sqrt{47}\,e^{-t} \sin\left(\tfrac{\sqrt{47}}{2}t\right) + \frac{-c_2}{2} e^{-\frac{t}{2}} \sin\left(\tfrac{\sqrt{47}}{2}t\right) + c_2 e^{-\frac{t}{2}} \cos\left(\tfrac{\sqrt{47}}{2}t\right)\frac{\sqrt{47}}{2}$$

$$y'(0) = e^0 \cos(0) - \sqrt{47}\,e^0 \sin(0) - c_2 e^0 \sin(0) + c_2 e^0 \cos(0)\frac{\sqrt{47}}{2} = 0$$

$$c_2 \cdot \frac{\sqrt{47}}{2} = -\frac{2}{\sqrt{47}} \qquad c_2 = -\frac{2}{\sqrt{47}}$$

$$y(t) = -2e^{-\frac{t}{2}} \cos\left(\tfrac{\sqrt{47}}{2}t\right) - \frac{2}{\sqrt{47}} e^{-\frac{t}{2}} \sin\left(\tfrac{\sqrt{47}}{2}t\right)$$