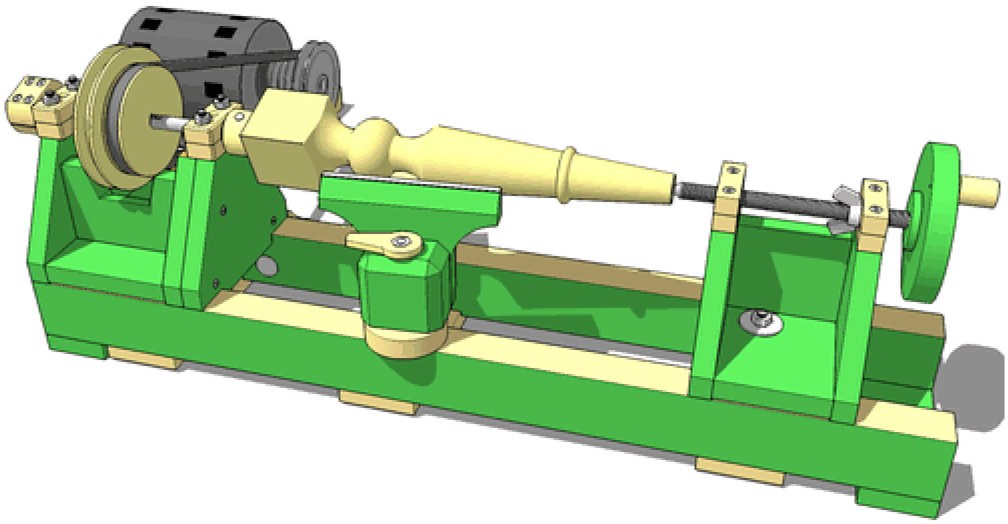
Modeling Solids of Revolution



A lathe creates a symmetrical object by rotating a solid piece of material and cutting away the edges to fit a particular shape. As the object is turning during the cutting, the object created is a solid of revolution.

This type of construction is a common process in manufacturing. We are going to model this process mathematically using paper party decorations. This is a convenient way to create a model for something we can then later make out of solid materials.

To complete this project, you will need:

- A paper party decoration (your instructor will provide several to choose from)
- A calculator with regression functions
- Graph paper
- Ruler
- Pencil

Note: Some of the party decorations are too elaborate to construct on a real lathe in a single operation as the shapes are not functions. We can still use this process to estimate the volume and surface area, but we would need a different manufacturing process to obtain the actual object.

Our first task is to obtain a function that models the shape of the curve. We will then use that function to obtain the volume and surface area.

Some information you will need before completing the project.

- As of November 2016, Acrylic resin prices for plastics are around \$0.05/lbs. And 1000 in^3 of resin is equivalent to around 42.9 lbs.
- Spray paint for the surface costs \$3.87 for 50 ft^2.

A. Choose the object you are going to model. Which object did you choose? Describe/sketch the object below.

B. Does the exterior surface appear to be a function?

C. Remove the included graph paper from the back of the packet (you can use more paper if the object you choose is too large for one sheet; if you do, however, be sure to securely attach the sheets together so that the gridlines align). Mark your axis of rotation on the graph paper, and line your object along the axis of rotation with one end at a gridline. Mark this point at the origin.

D. Trace your object on the graph paper. Measure your gridlines in millimeters. Clearly mark both the vertical and the horizontal axes along the gridlines, as well as the endpoint of your function where it meets the axis of rotation again.

E. Record the measurements you obtain from your trace at each gridline (and the endpoint) for the outer curve. Record the distance in the horizontal in the first column in the table below, and the corresponding height measurement in the second column.

Horizontal Distance	**Vertical Height**

You should have a minimum of 10 measurements, but depending on the size and complexity of your object, you may have more. If you need more room, add a separate sheet of paper. Be sure to include all data using in your final submission.

F. Record the measurements you obtain from your trace of any inset curves (if your cross-section is not a function) at each gridline (and the endpoint). Record the distance in the horizontal in the first column in the table below, and the corresponding height measurement in the second column. You should have at least three measurements for the upper and lower curves, including the beginning (or where they meet) and end of each curves.

Horizontal Distance	Vertical Height

Check with your instructor.

G. We will first try to find a regression equation for the curves. If your data is clearly piecewise, mark the beginning and ends of each piece, and perform the following analysis for each piece. You may find circles and ellipses work better this way also. For the first piece, enter your data into your calculator and obtain a model for your exterior curve following the steps below.

1. Enter data into the calculator by typing STAT and selecting **Edit**. Put the independent variable in **L1** and the dependent variable in **L2** (for the first data set, but you may want to use other lists for the next set of graphs so you still keep all the data). When finished entering data, 2nd MODE is the **Quit** function to escape back to the main screen.
2. To draw a scatterplot of the data, select 2nd Y= to enter **StatPlot**. To turn on **Plot1**, select it. Highlight **ON** and select it. Select the scatterplot option. Select the sources for your data (L1, L2, or whatever is appropriate). When all the fields are

completed, press ZOOM 9 for **ZoomStat** to automatically reset the axes to match your data.

3. To obtain regression equations, select STAT and scroll over to **Calc** menu. Select the appropriate regression equation. Options for linear, quadratic, cubic, quartic, exponential, power, natural logarithm, logistic and sine regression equations. Each function solves for coefficients and specifies what each represents in the equation. (Linear regression is modeled in one handout.) In each case, a correlation (r) or (R^2) value should be output along with the coefficients. Record this value along with the equation. You will want to experiment with different types of equations to model the data before choosing the best one (based on correlation and end behavior.

4. If you are using pieces, you may choose three or four curves you think are most likely for each section. If you are using a single curve, then try all the available models (logistic and sine regression do not provide R^2 values).

H.

Regression equation type	**Regression equation obtained**	r^2 *or* R^2 **(4 decimal places)**

I. Use the second table for your inset function if they exist. For these curves, with much less data, stick with about three simpler models for each curve.

Regression equation type	**Regression equation obtained**	r^2 *or* R^2 **(4 decimal places)**

J. Compare your equations to the model data in your calculator. Visually inspect each on the graph and using r^2 for each equation, determine which equation fits best with the least number of variables.

Give a sketch of your best-fit equation(s) compared to the data.

K. Which equation did you choose. Explain your reasoning. Describe how well you think it fits the data?

Check with your instructor.

L. Set up an integral to find the volume of the solid of revolution. Are you using the shell method or the washer/disk method? The Disk Method formula is $V = \pi \int_a^b [r(x)]^2 dx$ and the Shell Method formula is $V = 2\pi \int_a^b r(x)h(x)dx$. Sketch a graph of your process.

M. Calculate your integral by hand or in the calculator. Round your answer to four decimal places.

N. Calculate your result using your original measurements using Simpson's Rule or Trapezoidal Rule (depending on the number of data points you have). Recall the formulas are respectively:

$$\int_a^b f(x)dx \approx \frac{b-a}{3n}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 4f(x_{n-1}) + f(x_n)]\ or$$

$$\int_a^b f(x)dx \approx \frac{b-a}{2n}[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

O. Perform the same two calculations for any internal sections that will need to be carved out, and subtract the results to obtain your final values.

P. Which one do you think is more accurate? Why? Explain.

Check with your instructor.

Q. To calculate the cost of materials for our object, we need to convert the volume to cubic inches. Recall that 1 in = 2.54 cm.

R. How much will your finished object weigh? Use the material conversion information provided at the beginning of the project.

S. How much will your object cost in materials if made to the same size as the original?

Check with your instructor.

T. Use the same function to find the surface area of your object. Set up the integral. Recall that the surface area is $S = 2\pi \int_a^b f(x)\sqrt{1 + [f'(x)]^2}dx$ around the x-axis, and $S = 2\pi \int_a^b x\sqrt{1 + [f'(x)]^2}dx$ around the y-axis. Include all parts including carve-out sections.

U. Integrate with technology to obtain the total surface area.

V. Compare with Simpson's Rule or Trapezoidal Rule with the original data.

W. Choose the version you believe is more accurate.

X. Convert your area units to square feet.

Y. How much will it cost to paint your object? Use the information provided at the beginning of the project.

Check with your instructor.

Z. Suppose that you want to rescale your object (up or down) and make it out of different material. Research material costs and describe how you will rescale. Calculate the cost to produce your object with the new material and size.
Note: This item is open-ended, but list any sources for current market value of materials. We are not calculating manufacturing cost (since that requires machines and labor), only material costs.