MTH 154, Homework #1, Spring 2019 Name _____

Instructions: Work problems on a separate sheet of paper and attach work to this page. You should show all work to receive full credit for problems. Questions with compact answers can be recorded directly on this page. Graphs and longer answers that won't fit here, indicate which page of the work the answer can be found on and be sure to clearly indicate it on the attached pages. You may use Excel to complete the problems, but then submit Excel files online.

1. List all the elements of the indicated sets and answer the questions that follow. Recall that ∪ = union (OR), ∩ = intersection (AND), A' means "not A" or complement of A, and $n(A)$ is the number of elements in A.
   a. Let A be the set of all the letters in the title ROMEO AND JULIET.
   b. Let B be the set of all the letters in the title JULIUS CAESAR.
   c. Find $A \cup B$.
   d. $|A| = n(A)$
   e. If U is the set of all letters in the English alphabet, what is B'?
   f. Find $A \cap B$.
   g. $|A \cap B| = n(A \cap B)$

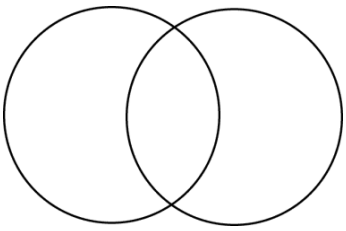
2. List the numbers in the sets.
   a. $\{x | x \text{ is a real number such that } x^2 = 1\}$
   b. $\{x | x \text{ is a positive integer less than } 12\}$
   c. $\{x | x \text{ is the square of an integer and } x < 100\}$
   d. $\{x | x \text{ is an integer such that } x^2 = 2\}$

3. Let $Q = \{x | x \text{ is an even counting number less than } 20\}$ and let $R = \{x |\ x \text{ is an integer divisible by } 2\}$. Determine whether the following statements are true or false. Recall that ∈ = "is an element of the set", and ⊂ is "subset of". [Hint: list out the elements of Q and R.]
   a. $Q \in R$
   b. $-4 \in Q$
   c. $6 \subset Q$
   d. $Q \subset R$
   e. $6 \in R$
   f. $\{6\} \subset R$

4. Let $A = \{1, 2, 3, 4, 5\}, B = \{0, 3, 6\}$. Find the following sets:
   a. A∪B
   b. A∩B

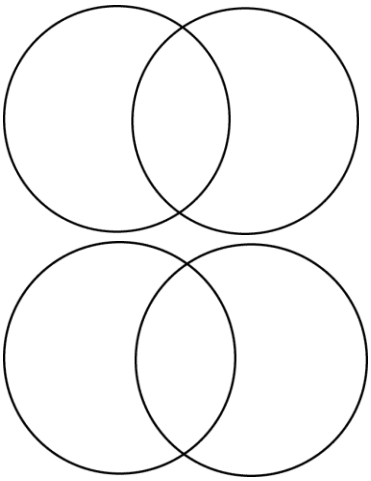
5. Use a Venn diagram to illustrate the relationship that A⊂B and B⊂C.

6. Use the Venn Diagrams below to draw the indicated sets.



   a. $A' \cap B$

b. $B - A$



c. $(A \cap B) \cup (A \cup B)'$

7. Recall that if $a \in A, b \in B$, then $A \times B$ is a set whose elements have the form (a, b). If $A = \{a, b, c, d\}, and\ B = \{y, z\}, and\ C = \{0,1\}$:
   a. List all the elements of A×B. What is the cardinality of the resulting set?
   b. List all the elements of A×B×C. What is the cardinality of the resulting set? [Hint: use your answer to part a.]
   c. List all the elements of A^2 (i.e. $A \times A$).

8. Let p and q be the propositions p: "I bought a lottery ticket", and q: "I won the million-dollar jackpot." Write each of the following compound propositions as English sentences. Recall that $\sim$ is "not", $\rightarrow$ is "if… then", $\vee$ = OR, and $\wedge$ = AND.
   a. $\sim p$
   b. $p \rightarrow q$
   c. $p \leftrightarrow q$
   d. $\sim p \wedge \sim q$
   e. $p \vee q$
   f. $p \wedge q$
   g. $\sim p \rightarrow \sim q$
   h. $\sim p \vee (p \wedge q)$

9. Let p and q be the propositions p: "You drive over 65 miles per hour", and q: "You get a speeding ticket." Write each of the following statement in terms of p, q, and logical connectives.
   a. You do not drive over 65 miles per hour.
   b. You drive over 65 miles per hour, but you do not get a speeding ticket.
   c. You will get a speeding ticket, if you drive over 65 miles per hour.
   d. If you do not drive over 65 miles per hour, you will not get a speeding ticket.
   e. Driving over 65 miles per hour is sufficient for getting a ticket.
   f. You get speeding ticket, but you do not drive over 65 miles per hour.
   g. Whenever you get a speeding ticket, you are driving over 65 miles per hour.

10. Determine the truth value for each statement. If the statement is false, find a counterexample. Recall that $\forall$ = "for all", $\exists$ = "there exists", ! = "unique", Z is integers, R is real numbers.
    a. $\forall n(n^2 \geq 0), n \in Z$
    b. $\exists n(n^2 = 2), n \in Z$
    c. $\exists !x(x + 3 = 2x), x \in R$
    d. $\exists x(x^2 = -1), x \in R$
    e. $\forall x(x^2 \neq x), x \in R$
    f. $\exists !x(x^2 = 1), x \in R$

11. Toss a thumbtack on a hard surface 50 times. How many times did it land with the point up? What is the approximate probability of landing point up?

12. You read in a book on poker that the probability of getting 3-of-a-kind in a 5-card poker hand is 1/47. Explain in plain language what this means in terms of proportions.

13. Ask several of your friends (at least 10 people) to choose a 4-digit number 'at random'. How many of the numbers chosen start with a 1 or 2? How many start with 8 or 9? (There is strong evidence that people in general tend to choose numbers starting with low digits.) Record all the numbers here.

14. The baseball player Ichiro Suzuki gets a hit about 33.1% of the time over an entire season. After he failed to hit safely in nine straight at-bats, the TV commentator says 'Ichiro is due for a hit by the law of averages.' IS that right? Why? This error is so common it has its own name: The Gambler's Fallacy.

15. The risk of dying in a fatal car crash is many times higher than the risk of dying in an airplane, yet many people are still more afraid of flying than driving to work every day. Explain why this represents a failure to understand probabilities/proportions. You may want to do research on the web for an explanation of the reasons why people might still think this despite the evidence.

16. An icosahedral die is a die with 20 sides that is sometimes used to play certain games. What is the proportion of successive rolls that have a certain face (assume they are numbered 1-20) will be face up?

17. A couple plans to have three children. There are 8 possible arrangements of girls and boys. For example, GGB means the first two children are girls and the third is a boy. All 8 arrangements are approximately equally likely. a) Write down all 8 arrangements of the sexes of three children. What is the proportion of any one of these arrangements? b) What is the proportion of outcomes such that the couple's children are 2 girls and a boy (not necessarily in that order)?

18. Use the Excel file **154data1.xlsx** to answer the questions that follow. Submit your Excel file to Blackboard even if you submit other written answers on paper.
    a. Use set notation to list the possible "Age" variable values.
    b. Use an IF function to find out whether the person is Gender = 2, then count the number of people for which the statement is true.
    c. Use the SUM formula to find out the total number of children represented in the data set.
    d. Create a table of States and display the counts. What proportion of the data set resides in Texas?
    e. Use an IF statement to determine if the salary value is less than $55,000. What proportion of the data set has a salary less than $55,000?