KEY

MTH 161, Practice Exam #2, Spring 2019

1. Is $x = -2$ a solution to the inequality $\frac{2-3x}{x-2} > 0$? Explain. it is not.

2. The inequality $x^2 + 1 < 0$ has no real solution. Explain why. x^2+1 always >0

3. Give an example of a rational function with a vertical asymptote at $x = 1$ and a horizontal asymptote at $y = 2$.
$f(x) = \frac{2x+3}{x-1}$

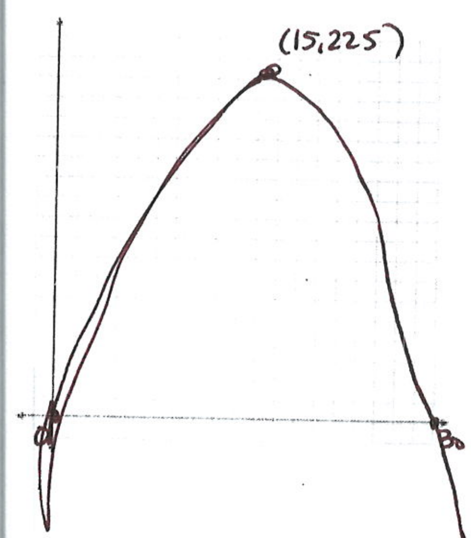
4. Use the intermediate value theorem to show that the function $f(x) = -2x^2 + 5x + 11$ has a real zero on the interval [2,4]. $f(2) = 13$, $f(4) = -1$ f is continuous.

5. Find the partial fraction decomposition of $\frac{2x-1}{x^2-4x-12}$. $= \frac{11/8}{x-6} + \frac{5/8}{x+2}$

6. Given the following polynomial function, $f(x) = \frac{1}{2}(x-1)^3(x+3)^2(x^2+4)$.
   a. Identify the real zeros and multiplicity. 1 mult. 3, -3 mult 2, $\pm 2i$ imaginary
   b. How does the function behave at each zero (touch, cross, etc.) 1 crosses w/ kink
   -3 touches
   c. The graph behaves like the function $y =$ $\frac{1}{2}x^7$ for large values of $|x|$.

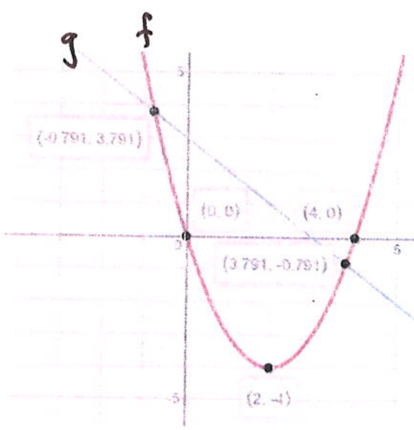
7. Use long division to find $\frac{x^5-4x^3+x^2+1}{x^2-2x-3}$. Write your final answer as $q(x) + \frac{r(x)}{d(x)}$.
$x^3+2x^2+3x+9 + \frac{27x+28}{x^2-2x-3}$

8. A toy store has 30 meters of fencing to fence off a rectangular area for an electric train display in one corner of the store. Two sides are against the wall and will need no fence.
   a. Write an equation that represents the total length of fence in terms of W and L. (Let $L = x$.)
   $A = xy = WL$
   b. Write an equation that represents the area of $A(x)$.
   $A(x) = 30x - x^2$
   c. Find the maximum area and give the dimensions.
   max @ (15,225) 15x15
   d. Sketch the graph of $A(x)$, label and use an appropriate domain and range.
   D: [0,30] R: [0,225]



9. Solve the inequalities, showing sign charts for each.
   a. $2x(x-1)^2(3-x) < 0$ $(-\infty,0)\cup(3,\infty)$
   b. $\frac{3-x}{x+1} \geq 2$ $(-1, 1/3]$

10. Consider the graph $f(x)$ and $g(x)$ to the right.



a. $f(x) = g(x)$ $(-0.791, 3.791)$ & $(3.791, -0.791)$ or $x = -0.791$ & $x = 3.791$

b. $f(x) < g(x)$ $x \in (-0.791, 3.791)$

c. $f(x) = 0$ $x=0,\ x=4$

11. Consider the polynomial function

$$f(x) = x^5 + 3x^4 - 9x^3 - 21x^2 - 10x - 24$$

a. Based on degree, how many real zeros and complex zeros does f have? 5

b. Using the Rational Zeros Theorem, list the possible rational zeros of f.
$\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

c. Use synthetic division to show that $x = -2$ is a zero of f.
$(x+2)(x^4 + x^3 - 11x^2 + x - 12)$

d. Finish factoring the polynomial. Write as a product of linear factor with complex numbers where appropriate.
$(x+2)(x+4)(x-3)(x-i)(x+i)$

e. List all real and imaginary zeros of the function.
$-2, -4, 3, \pm i$

12. Consider the rational function $f(x) = \frac{9-3x}{2+x}$. Find the following properties of the function. If it does not apply, write DNE.

a. Holes DNE

b. Vertical asymptote(s) $x=-2$

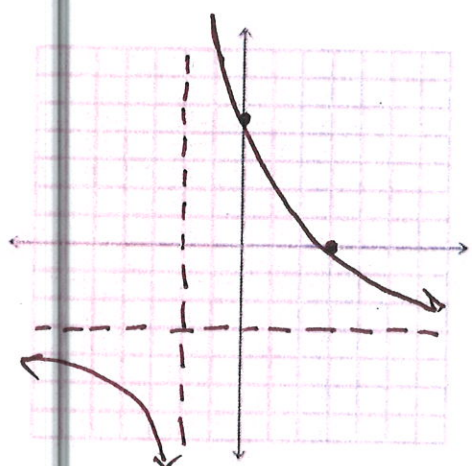
c. Horizontal asymptote $y=-3$

d. Domain $(-\infty,-2) \cup (-2,\infty)$

e. X-intercept(s) $x=3$ $(3,0)$

f. Y-intercept $y=\frac{9}{2}$ $(0, 9/2)$

g. Sketch the graph, labeling each element from the list above.



13. What value should you wright in the circle to check whether $(x + 4)$ is a factor of $f(x) = x^3 - 2x^2 + 3x + 4$?

$$\begin{array}{r|rrrr} -4 & 1 & -2 & 3 & 4 \\ \hline \end{array}$$

14. What feature of the graph of $y = \frac{5}{x-3}$. What can you find by solving $x - 3 = 0$? Vertical asymptote

15. Is $y = \frac{2}{3}$ a horizontal asymptote of $y = \frac{2x}{3x^2-9}$. no, 0 is

16. If a zero of a polynomial f is of odd multiplicity, then the graph of f crosses the x-axis at that zero.

17. Suppose that $-2 + i$ is a zero of a polynomial function. This implies that $-2 - i$ is also a zero.

18. If $x = -3$ is a zero of a polynomial function f, then $(x+3)$ is a factor of the polynomial $f(x)$.

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work

1. $\frac{2-3(-2)}{-2-2} = \frac{2+6}{-4} = \frac{8}{-4} = -2 \overset{?}{>} 0$ no

4. $f(2) = -2(2)^2 - 5(2) + 11 = -8+10+11 = 13$

$f(4) = -2(4)^2 + 5(4) + 11 = -32+20+11 = -1$

Since there is a sign change and function is continuous there must be a zero in interval

5. $\frac{2x-1}{x^2-4x-12} = \frac{A}{x-6} + \frac{B}{x+2} = \frac{A(x+2)+B(x-6)}{x^2-4x-12}$

$(x-6)(x+2)$

Method 1: $A(x+2) + B(x-6) = 2x-1$

if $x=6$ $\quad 8A + 0 = 12-1$

$8A = 11 \Rightarrow A = 11/8$

if $x=-2$ $\quad 0 - 8B = -4-1$

$-8B = -5 \Rightarrow B = 5/8$

Method 2:

$Ax + 2A + Bx - 6B = 2x-1$

$\therefore$ $A+B=2 \rightarrow B = 2-A$

$2A-6B=-1$

$2A - 6(2-A) = -1$

$2A - 12 + 6A = -1$

$8A = 11$

$A = 11/8$

$B = 2 - 11/8$

$= \frac{16}{8} - \frac{11}{8} = \frac{5}{8}$

$\frac{2x-1}{x^2-4x-12} = \frac{11/8}{x-6} + \frac{5/8}{x+2}$

7. $x^2-2x-3 \overline{)\, x^5+0x^4-4x^3+x^2+0x+1}$ quotient x^3+2x^2+3x+9

$-\cancel{x^5}+2x^4+3x^3$

$\cancel{2x^4}-x^3+x^2+0x+1$

$-\cancel{2x^4}+4x^3+6x^2$

$\cancel{3x^3}+7x^2+0x+1$

$-\cancel{3x^3}+2x^2+9x$

$\cancel{9x^2}+9x+1$

$-\cancel{9x^2}+18x+27$

$27x+28$

8a.

$x + y = 30 \rightarrow 30 - x = y$

$A = xy \rightarrow A(x) = x(30 - x) =$
$30x - x^2$

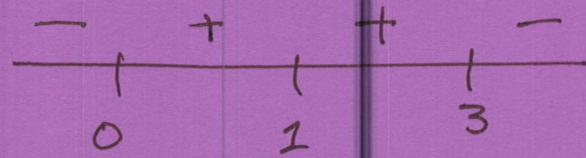
9\. a. $2x(x-1)^2(3-x) < 0$

$x = -1$
$(-)(-1)^2(+)$

$x = 1/2$
$(+)(-1)^2(+)$

$x = 4$
$(+)(+)^2(-)$

$x = 2$
$(+)(+)^2(+)$



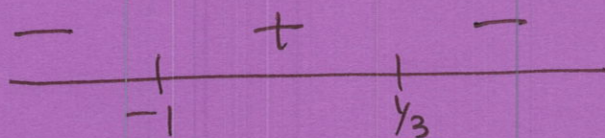
$(-\infty, 0) \cup (3, \infty)$

b. $\frac{3-x}{x+1} \geq 2 \rightarrow \frac{3-x}{x+1} - 2\frac{(x+1)}{(x+1)} \geq 0 \rightarrow \frac{3-x-2x-2}{x+1} \geq 0$

$\rightarrow \frac{-3x+1}{x+1} \geq 0$

$-3x + 1 = 0$
$x = 1/3$

$x + 1 = 0$
$x = -1$



$x = 0$ $\frac{+}{+}$

$(-1, 1/3]$

11c.

$$\begin{array}{r|rrrrrr} -2 & 1 & 3 & -9 & -21 & -10 & -24 \\ & & -2 & -2 & 22 & -2 & 24 \\ \hline & 1 & 1 & -11 & 1 & -12 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} -4 & 1 & 1 & -11 & 1 & -12 \\ & & -4 & 12 & -4 & 12 \\ \hline & 1 & -3 & 1 & -3 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 1 & -3 & 1 & -3 \\ & & 3 & 0 & 3 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$(x+2)(x+4)(x-3)(x^2+1)$
$(x-i)(x+i)$

12. $f(x) = \frac{3(3-x)}{2+x}$

b. $2+x=0 \Rightarrow x=-2$

c. $\frac{-3x}{x} = -3 = y$

d. $(-\infty,-2) \cup (-2,\infty)$

e. $3-x=0 \quad x=3$

f. $\frac{9}{2} = y$

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