KEY

MTH 161, Practice Final Exam, Spring 2019

1. Write the system of equations represented by the augmented matrix
$$\left[\begin{array}{ccc|c} 2 & -1 & 8 & 1 \\ 3 & 0 & 3 & -1 \\ -1 & -3 & 0 & 5 \end{array}\right]$$
$$\begin{cases} 2x - y + 8z = 1 \\ 3x \quad + 3z = -1 \\ -x - 3y \quad = 5 \end{cases}$$

2. Explain the meaning of $-3R_1 + R_2 \rightarrow R_2$

multiply Row1 by (−3) add the result to Row 2, then put back in matrix in place of Row 2

3. Suppose that the x-intercepts of $f(x) = y$ are 7 and -2. What are the x-intercepts of the graph of $-f(x+2)$.
5, −4

4. For the system of equations $\begin{cases} x - 2y + 3z = 7 \\ 3y \quad = 5 \\ 2z \quad = -1 \end{cases}$
$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 0 & 3 & 0 & 5 \\ 0 & 0 & 2 & -1 \end{array}\right]$$

   a. Write as an augmented matrix.

   b. What is the dimension of the resulting matrix? 3x4

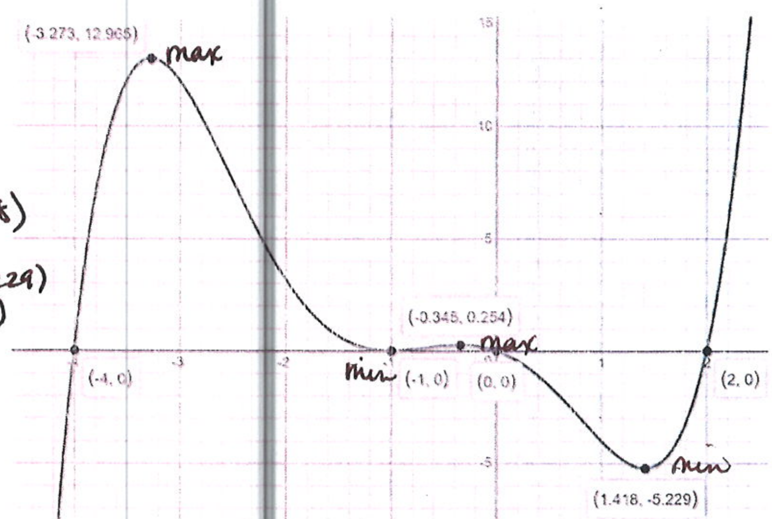
   c. Solve the system. $x = 71/6$, $y = 5/3$, $z = -1/2$

5. Consider the matrix $\left[\begin{array}{ccc|c} 1 & -1 & 2 & 6 \\ 0 & 1 & 5 & -12 \\ 0 & 4 & 2 & -10 \end{array}\right]$

   a. Perform row operations to put the matrix in reduced row echelon form.

   b. Solve the system. $x = 79/9$, $y = -13/9$, $z = -19/9$

6. Given the graph of $f(x)$ shown,



   a. Find the intervals on which f is increasing. $(-\infty, -3.273) \cup (1.418, \infty) \cup (-1, -0.345)$

   b. Where is it decreasing? $(-3.273, -1) \cup (-0.345, 1.418)$

   c. State any relative minima or maxima. min $(-1, 0)$, $(1.418, -5.229)$
   max $(-3.272, 12.965)$ & $(-0.345, 0.254)$

7. The daily revenue R in dollars achieved by selling x boxes of cookies is found to be $R(x) = 6.5x - 0.03x^2$. The daily cost to produce x boxes of cookies is found to be $C(x) = 1.10x + 150$.

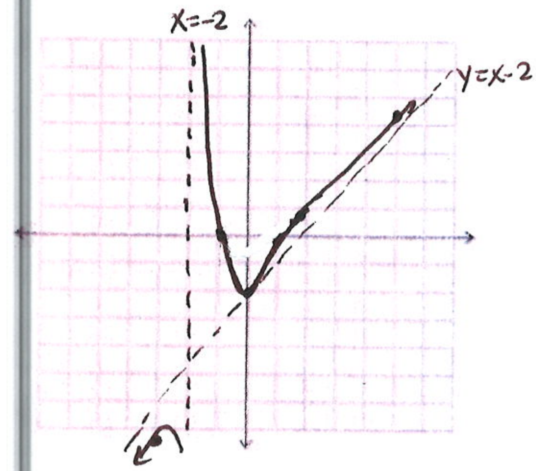
   a. Profit = Revenue – Cost. What is the profit function? Simplify completely.
   $P(x) = -0.03x^2 + 5.4x - 150$

   b. How many boxes of cookies will need to be produced to maximize profit? What is the maximum profit? (Warning: boxes must be an integer.)

   boxes $x = 90$ profit max 93

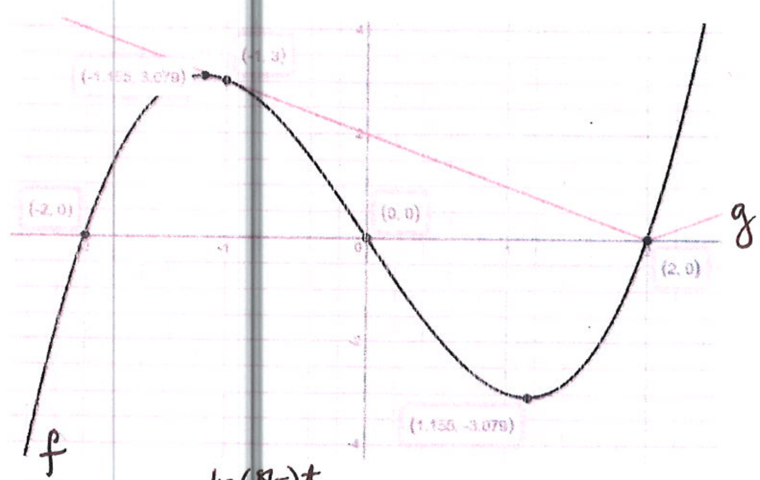
8. Given the polynomial $f(x) = x^5 + 6x^4 - 24x^2 - \quad x - 30$,
   a. List all possible rational zeros of f by using the rational zeros theorem.
   $\pm1, \pm2, \pm3, \pm5, \pm6, \pm10, \pm15, \pm30$
   b. Factor $f(x)$ as a product of linear factors using any method.
   $(x+5)(x+3)(x-2)(x-i)(x+i)$
   c. List all zeros of $f(x)$ both real and imaginary.
   $-5, -3, 2, \pm i$

9. Given the rational function $f(x) = \frac{x^2-1}{x+2}$
   a. Find the x- and y-intercepts
   x int: ±1, y-int: -2
   b. Write the equation of any vertical, horizontal or slant asymptotes. VA: $x=-2$ SA: $y = x-2$
   c. Evaluate $f(2), f(-3), f(5)$
   $\frac{3}{4}$ -8 $\frac{24}{7}$
   d. Using this information, sketch an accurate graph of $f(x)$ by hand on axes shown. Label all information from the parts above.



10. Solve the equations algebraically. Answers should be intervals, exact values or expressions.
   a. $\sqrt[3]{x-4} + 5 = -1$ $x = -212$
   b. $3(2^{5x}) = 48$ $x = 4/5$
   c. $\frac{(x-3)^2}{x^2-1} \geq 0$ $(-\infty, -1) \cup (1, \infty)$
   d. $\log_4 x + \log_4(x-10) = 2$ $x = 5 + \sqrt{41}$

11. Use the graph of $f(x)$ and $g(x)$ shown to answer the following.
   a. $g(x) = 0$ 2
   b. $f(x) > 0$ $(-2, 0) \cup (2, \infty)$
   c. $f(x) = g(x)$ $x = -1, 2$



12. The population of a colony of fire ants obeys the law of uninhibited growth following the model $A(t) = A_0 e^{kt}$. If there are 500 ants initially and 800 ants after 1 week.
   a. Solve for A_0 and k for the model.
   $A(t) = 500e^{\ln(8/5)t}$
   b. What is the size of the colony after 3 weeks?
   2048
   c. After how many days will there be 5000 ants?
   4.9 weeks

13. Given that $x - c$ divides evenly into $f(x)$, which statements are true (check all).

a. $(x - c)$ is a factor of $f(x)$

b. $-c$ is a zero of $f(x)$

c. The remainder of dividing $f(x)$ by $x - c$ is 0

d. The y-intercept of $f(x)$ is c

e. One x-intercept of $f(x)$ is c

14. The graph of $f(x) = \frac{3x^2+2}{5x^2}$ will behave like which function for large values of $|x|$? $y = \frac{3}{5}$

15. Given the piecewise function $f(x) = \begin{cases} 5 - 2x, & x > -1 \\ x^2, & x \leq -1 \end{cases}$, what is $f(-1)$? $f(-1) = 1$

16. Which statement(s) is(are) equivalent to $\log_2 \frac{1}{64} = -6$

a. $2^{1/6} = 64$

b. $-\frac{\ln(64)}{\ln 2} = -6$

c. $2^{-6} = \frac{1}{64}$

d. $(-6)^2 = \frac{1}{64}$

17. Is the statement true or false: The function $f(x) = x^3 \ln(4 - x^2)$ is odd. true

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work

4c. $\left[\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 0 & 3 & 0 & 5 \\ 0 & 0 & 2 & -1 \end{array}\right]$ $\to \frac{1}{2}R_3 \to R_3$, $\to \frac{1}{3}R_2 \to R_2$ $\left[\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 0 & 1 & 0 & 5/3 \\ 0 & 0 & 1 & -1/2 \end{array}\right]$

$-3R_3 + R_1 \to R_1$ $\left[\begin{array}{ccc|c} 1 & -2 & 0 & 17/2 \\ 0 & 1 & 0 & 5/3 \\ 0 & 0 & 1 & -1/2 \end{array}\right]$ $2R_2 + R_1 \to R_1$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 71/6 \\ 0 & 1 & 0 & 5/3 \\ 0 & 0 & 1 & -1/2 \end{array}\right]$

5a. $\left[\begin{array}{ccc|c} 1 & -1 & 2 & 6 \\ 0 & 1 & 5 & -12 \\ 0 & 4 & 2 & -10 \end{array}\right]$ $-4R_2 + R_3 \to R_3$ $\left[\begin{array}{ccc|c} 1 & -1 & 2 & 6 \\ 0 & 1 & 5 & -12 \\ 0 & 0 & -18 & 38 \end{array}\right]$

$\frac{-1}{18}R_3 \to R_3$ $\left[\begin{array}{ccc|c} 1 & -1 & 2 & 6 \\ 0 & 1 & 5 & -12 \\ 0 & 0 & 1 & -19/9 \end{array}\right]$ $-5R_3 + R_2 \to R_2$, $-2R_3 + R_1 \to R_1$ $\left[\begin{array}{ccc|c} 1 & -1 & 0 & 92/9 \\ 0 & 1 & 0 & -13/9 \\ 0 & 0 & 1 & -19/9 \end{array}\right]$

$R_1 + R_2 \to R_1$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 79/9 \\ 0 & 1 & 0 & -13/9 \\ 0 & 0 & 1 & -19/9 \end{array}\right]$

7\. a. $6.5x - 0.03x^2 - (1.10x + 150) = -0.03x^2 + 5.4x - 150 = P(x)$

b. $x = 90,\ y = 93$

9\. b. $x + 2 = 0$
$x = -2$

$$\begin{array}{r} x - 2 \\ x+2 \overline{)\, x^2 + 0x - 1} \\ -x^2 \mp 2x \\ \hline -2x - 1 \\ +2x + 4 \\ \hline 3 \end{array}$$

a. $x^2 - 1 = 0 \to x = \pm 1$

$\frac{0-1}{0+2} = -2$

c. $\frac{2^2-1}{2+2} = \frac{3}{4}$ $\frac{9-1}{-3+2} = \frac{8}{-1} = -8$

$\frac{25-1}{5+2} = \frac{24}{7}$

10\. a. $\sqrt[3]{x-4} + 5 = -1$
$-5 \quad -5$
$\sqrt[3]{x-4} = -6$
$x - 4 = -216$
$+4 \quad +4$
$x = -212$

b. $3(2^{5x}) = 48$
$2^{5x} = 16 = 2^4$
$5x = 4$
$x = 4/5$

c. $x^2 - 1 = 0$ $\frac{(x-3)^2}{(x-1)(x+1)}$
$x = \pm 1$
$(x-3)^2 = 0$ $x = 3$

$+ \quad - \quad + \quad +$
$-1 \quad 1 \quad 3$

$x = -2$ $\frac{(-)^2}{(-)(-)}$ $x = 0$ $\frac{(-)^2}{(-)(+)}$

$x = 2$ $\frac{(-)^2}{(+)(+)}$ $x = 4$ $\frac{(+)^2}{(+)(+)}$

d. $\log_4 x + \log_4(x-10) = 2$
$\log_4[(x)(x-10)] = 2 \to x^2 - 10x = 16$ $= 2^4$
$x^2 - 10x - 16 = 0$ $x = \frac{+10 \pm \sqrt{100+64}}{2} = 5 \pm \sqrt{41}$
no neg.

(2)

12. a. $500e^{k(1)} = 800$

$e^{k} = \frac{8}{5} \rightarrow k = \ln(8/5)$

$A(t) = 500e^{(\ln 8/5)t}$

b. $A(3) = 2048$

c. $5000 = 500e^{(\ln 8/5)t}$

$10 = e^{(\ln 8/5)t}$

$\frac{\ln 10}{\ln 8/5} = t \approx 4.9$