Instructions: This project uses online graphing technology (specifically at GraphFree.com, although you may use any website that can graph piecewise functions that you prefer) to explore piecewise functions, their limits and continuity. Follow the directions below using a provided example. Submit a Word document with screenshot/images and your explanations to Blackboard by the posted due date.

For this project, we will be using the piecewise graphing program at GraphFree.com. Select the Start Graphing button, and you will find a graphing input screen somewhat similar to a graphing calculator. In the dropdown menu in the middle of the screen, select Piecewise.



Once you select Piecewise the screen will update to display multiple boxes for entering the pieces of your piecewise graph along with the inequalities to define where each piece is defined. If you need more than three pieces, you can go to the other optional plots and enter additional pieces. They will all graph on the same screen.

To graph the function


Show open and closed endpoint symbols

$f(x)=\left\{\begin{array}{ll}2 x+1, & \text { for } x<1 \\ x, & \text { for } x \geq 1\end{array}\right.$, we enter the pieces and select the appropriate inequalities from the dropdown menu. In the middle of the screen, you can adjust the display options such as the maximum and minimum values for each axis, the color and tick labels and other features. Then select Create Plot.

When the screen reloads, it will display the graph at the top of the screen. This function produces the following graph.

From this graph, you will then be able to answer questions like $\lim _{x \rightarrow 1^{-}} f(x)=3$ and $\lim _{x \rightarrow 1^{+}} f(x)=1$, $f(1)=1, \lim _{x \rightarrow 1} f(x)$ is undefined since the one-sided limits don't agree, the graph is discontinuous at $x=1$ since the limit doesn't exist, and this is a jump discontinuity and is, therefore, not removable.

For each of the piecewise functions below, construct a graph and then use the graph to answer the following questions for each function.


For each value $c$ where a piece of the graph changes find:
a. $\lim _{x \rightarrow c^{-}} f(x)$
b. $\lim _{x \rightarrow c^{+}} f(x)$
c. $f(c)$
d. $\lim _{x \rightarrow c} f(x)$
e. Is the function continuous at $x=c$ ?
f. If $f(x)$ is discontinuous at $x=c$, what kind of discontinuity is it? Removable or a jump? If the discontinuity is removable, what value should $f(c)$ be defined to be to repair the discontinuity?

Some of the graphs will have more than one break, and so you may need to answer each of these questions at two values.

1. $f(x)=\left\{\begin{array}{l}-x+4, \text { for } x<3 \\ x-3, \text { for } x \geq 3\end{array}\right.$
2. $f(x)=\left\{\begin{array}{l}x^{2}, \text { for } x<-1 \\ x+2, \text { for } x \geq-1\end{array}\right.$
3. $f(x)=\left\{\begin{array}{c}x+1, \text { for } x<0 \\ 2, \text { for } 0 \leq x<1 \\ 3-x, \text { for } x \geq 1\end{array}\right.$
4. $f(x)=\left\{\begin{array}{l}\frac{1}{2} x+1, \text { for } x<4 \\ -x+7, \text { for } x \geq 4\end{array}\right.$
5. $f(x)=\left\{\begin{array}{l}\frac{1}{x-3}, \text { for } x \neq 3 \\ 2, \quad \text { for } x=3\end{array}\right.$
