MTH Homework #4 Key

1a. $f(x) = x^3 - 3x + 6 \quad [-1, 3]$

$f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 0 \quad x = \pm 1$

$f(-1) = 8, \quad f(1) = 4, \quad f(3) = 24$

abs. min abs. max

b. $g(x) = (x+3)^{2/3} - 5 \quad [-4, 5]$

$g'(x) = \frac{2}{3}(x+3)^{-1/3}$ undefined at $x = -3$

$f(-4) = -4 \quad f(-3) = -5 \quad f(5) = -1$

abs. min absolute max

c. $h(x) = \frac{4x}{x^2+1} \quad [-3, 3]$

$h'(x) = \frac{4(x^2+1) - 4x(2x)}{(x^2+1)^2} = \frac{4x^2 + 4 - 8x^2}{(x^2+1)^2} = \frac{4 - 4x^2}{(x^2+1)^2} = \frac{4(1-x^2)}{(x^2+1)^2} \quad x = \pm 1$

$f(-3) = -1.2 \quad f(-1) = -2 \quad f(1) = 2 \quad f(3) = 1.2$

abs. min abs. max

2. See attached absolute max at (1,3). There is no absolute minimum

3. $R(x) = xp = 150x - \frac{1}{2}x^2 \qquad P(x) = 150x - \frac{1}{2}x^2 - (4000 + \frac{1}{4}x) =$

$C(x) = 4000 + \frac{1}{4}x \qquad 149.75x - \frac{1}{2}x^2 - 4000$

$P'(x) = 149.75 - x = 0 \quad x = 149.75$

$\Rightarrow$ round to 150 (higher profit than 149)

Profit max is \$7212.5

price $= 150 - \frac{1}{2}(150) = 150 - 75 =$ \$75

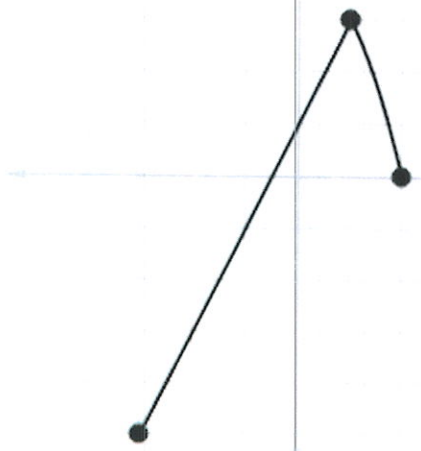
4. $2yy' - y - xy' + 2x = 0$

$2yy' - xy' = y - 2x$

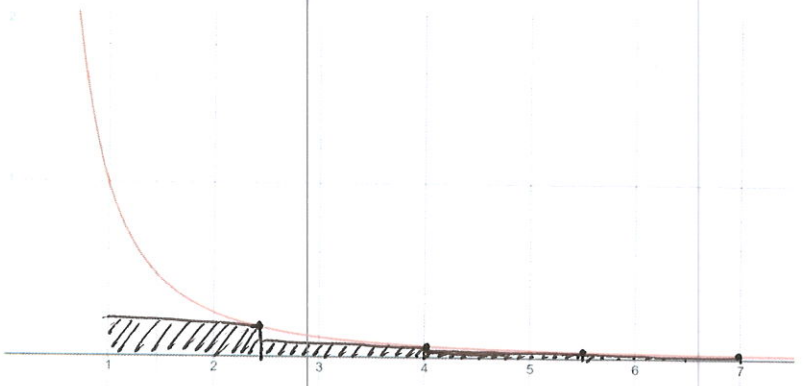
$y'(2y - x) = y - 2x \rightarrow y' = \frac{y - 2x}{2y - x}$

5a. $\Delta x = \frac{7-1}{4} = \frac{6}{4} = \frac{3}{2} \qquad \{1, 5/2, 4, 11/2, 7\} \qquad f(x) = \frac{1}{x^2}$

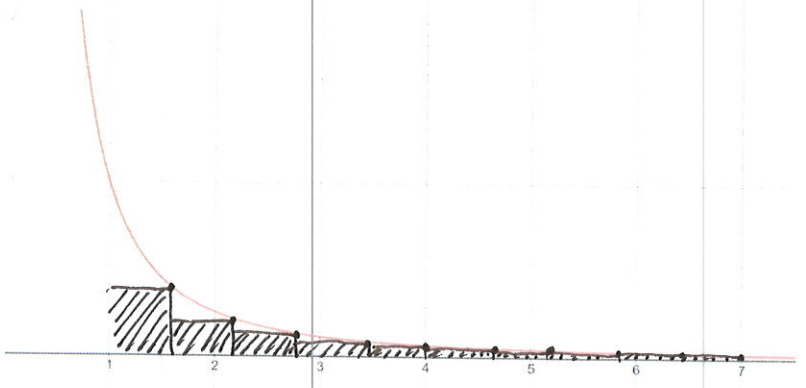
$\sum_{i=1}^{4} f(x_i)\Delta x = \frac{3}{2}\left[\frac{1}{(5/2)^2} + \frac{1}{4^2} + \frac{1}{(11/2)^2} + \frac{1}{7^2}\right] = 0.4139$



5a



5b.



5b. $\Delta x = \frac{7-1}{10} = \frac{6}{10} = \frac{3}{5} = .6$ $\{1, 1.6, 2.2, 2.8, 3.4, 4, 4.6, 5.2, 5.8, 6.4, 7\}$ $f(x) = \frac{1}{x^2}$

$$\sum_{i=1}^{10} f(x_i)\Delta x = \frac{3}{5}\left[\frac{1}{1.6^2} + \frac{1}{2.2^2} + \frac{1}{2.8^2} + \frac{1}{3.4^2} + \frac{1}{4^2} + \frac{1}{4.6^2} + \frac{1}{5.2^2} + \frac{1}{5.8^2} + \frac{1}{6.4^2} + \frac{1}{7^2}\right] = 0.6195$$

5c. $\Delta x = \frac{5-0}{6} = \frac{5}{6}$ $\{0, \frac{5}{6}, \frac{10}{6}, \frac{15}{6}, \frac{20}{6}, \frac{25}{6}, 5\}$ $f(x) = x^2+1$

$$\sum_{i=0}^{5} f(x_i)\Delta x = \frac{5}{6}\left[(0^2+1) + \left(\frac{5}{6}\right)^2+1 + \left(\frac{10}{6}\right)^2+1 + \left(\frac{15}{6}\right)^2+1 + \left(\frac{20}{6}\right)^2+1 + \left(\frac{25}{6}\right)^2+1\right] = 36.8287$$

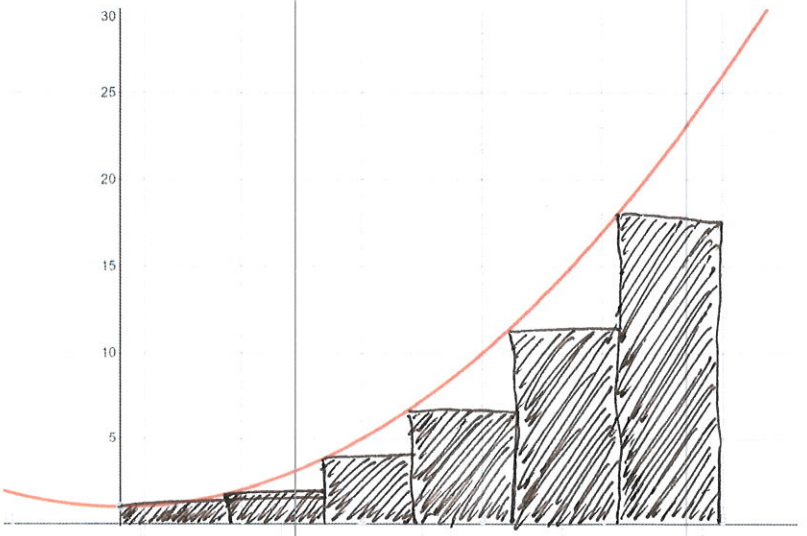
5d. $\Delta x = \frac{2-1}{5} = \frac{1}{5} = 0.2$ $\{1, 1.2, 1.4, 1.6, 1.8, 2\}$ $f(x) = e^x+1$

$$\sum_{i=0}^{4} f(x_i)\Delta x = 0.2\left[e^1+1 + e^{1.2}+1 + e^{1.4}+1 + e^{1.6}+1 + e^{1.8}+1\right] = 5.2193$$

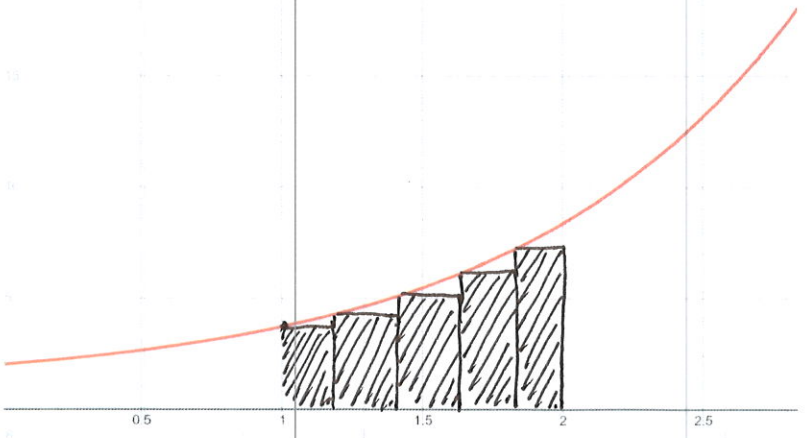
5e. $\Delta x = \frac{8-3}{3} = \frac{5}{3}$ $\{3, \frac{14}{3}, \frac{19}{3}, 8\}$ $f(x) = \ln(x^3-1)$

midpts $\{\frac{23}{6}, 5.5, \frac{43}{6}\}$

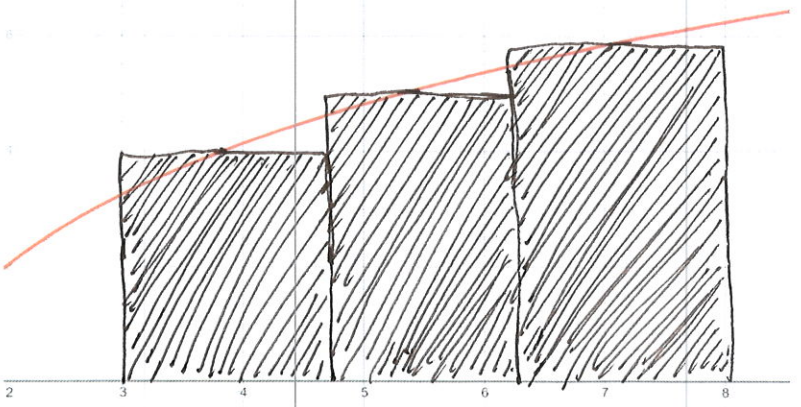
$$\sum_{i=1}^{3} f(x_i)\Delta x = \frac{5}{3}\left[\ln\left(\left(\frac{23}{6}\right)^3-1\right) + \ln\left((5.5)^3-1\right) + \ln\left(\left(\frac{43}{6}\right)^3-1\right)\right] = 25.0452$$



5c.



5d.



5e.