

Instructions: You must show all work to receive full credit for the problems below. You may check your work with a calculator, but answers without work will receive minimal credit. Use exact answers unless the problem starts with decimals or you are specifically asked to round.

1. Find the equation of the tangent line to the implicitly defined function $x^2y - 2x^3 - y^3 + 1 = 0$ at the point $(2, -3)$.

$$2xy + x^2y' - 6x^2 - 3y^2y' = 0$$

$$2xy - 6x^2 = 3y^2y' - x^2y' = y'(3y^2 - x^2)$$

$$\frac{2xy - 6x^2}{3y^2 - x^2} = y'$$

$$\frac{2 \cdot 2 \cdot (-3) - 6(2)^2}{3(-3)^2 - 2^2} = \frac{-12 - 24}{27 - 4} = \frac{-36}{23}$$

$$y + 3 = \frac{-36}{23}(x - 2)$$

2. Certain chemotherapy dosages depend on a patient's surface area. According to the Mosteller model, $S = \frac{\sqrt{hw}}{60}$, where h is the patient's height in centimeters, w is the patient's weight in kilograms, and S is the approximate surface area in square meters. Assume that Tom's height is a constant 165 cm, but he is on a diet. If he loses 2 kg per month, how fast is his surface area decreasing at the instant he weighs 70 kg?

$$\frac{dS}{dt} = \frac{\frac{1}{2} \frac{\sqrt{w}}{\sqrt{h}}}{60} \frac{dh}{dt} + \frac{\frac{1}{2} \frac{\sqrt{h}}{\sqrt{w}}}{60} \frac{dw}{dt}$$

$$= \frac{\frac{\sqrt{70}}{\sqrt{165}} (0) + \frac{\sqrt{165}}{\sqrt{70}} (2)}{120} = \frac{\sqrt{165}}{60\sqrt{70}} \approx 0.25588 \text{ cm}^2$$