

Instructions: You must show all work to receive full credit for the problems below. You may check your work with a calculator, but answers without work will receive minimal credit. Use exact answers unless the problem starts with decimals or you are specifically asked to round.

1. For the matrix $A = \begin{bmatrix} 1 & 1 & 0 & -1 \\ -2 & 1 & 0 & 0 \\ 3 & 2 & 0 & -1 \\ -1 & 0 & 1 & 1 \end{bmatrix}$, determine the following:

- The Rank of the matrix.
- The dimensions of $Nul A$.
- The dimensions of $Row A$.
- The dimensions of $Nul A^T$.
- The rank of A^{-1} if it exists.

2. For a 9×5 matrix with three pivots find:

- Dimensions of $Nul A$
- Dimensions of $Col A$
- Is the matrix one-to-one?
- Is the matrix onto?
- What are the dimensions of the vector space A maps from?
- What are the dimensions of the vector space A maps into?

3. Given the bases $B = \{b_1, b_2, b_3\}$ and $C = \{c_1, c_2, c_3\}$ below, find the change of basis matrices $P_{C \leftarrow B}$ and $P_{B \leftarrow C}$. If the B-coordinate vector for \vec{x} is as shown, find the C-coordinate vector for \vec{x} .

$$\vec{b}_1 = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \vec{b}_2 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}, \vec{b}_3 = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \vec{c}_1 = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \vec{c}_2 = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \vec{c}_3 = \begin{bmatrix} 0 \\ -1 \\ -2 \end{bmatrix}, [\vec{x}]_B = \begin{bmatrix} 1 \\ 0 \\ -9 \end{bmatrix}$$

4. For the vectors $\vec{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, and $\vec{v} = \begin{bmatrix} -6 \\ 9 \end{bmatrix}$, find the following:

a. $\|\vec{u}\|$

b. $\vec{u} \cdot \vec{v}$

c. Are \vec{u} and \vec{v} orthogonal?