MTH 266, Quiz #10, Spring 2019 Name KEY

Instructions: You must show all work to receive full credit for the problems below. You may check your work with a calculator, but answers without work will receive minimal credit. Use exact answers unless the problem starts with decimals or you are specifically asked to round.

1. For the matrix $A = \begin{bmatrix} 1 & 1 & 0 & -1 \\ -2 & 1 & 0 & 0 \\ 3 & 2 & 0 & -1 \\ -1 & 0 & 1 & 1 \end{bmatrix}$, determine the following:
   a. The Rank of the matrix. 4
   b. The dimensions of $Nul\ A$. 0
   c. The dimensions of $Row\ A$. 4
   d. The dimensions of $Nul\ A^T$. 0
   e. The rank of A^{-1} if it exists. 4

2. For a 9×5 matrix with three pivots find:
   a. Dimensions of $Nul\ A$ 2
   b. Dimensions of $Col\ A$ 3
   c. Is the matrix one-to-one? no
   d. Is the matrix onto? no
   e. What are the dimensions of the vector space A maps from? $\mathbb{R}^5$
   f. What are the dimensions of the vector space A maps into? $\mathbb{R}^9$

3. Given the bases $B = \{b_1, b_2, b_3\}$ and $C = \{c_1, c_2, c_3\}$ below, find the change of basis matrices $\underset{C \leftarrow B}{P}$ and $\underset{B \leftarrow C}{P}$. If the B-coordinate vector for $\vec{x}$ is as shown, find the C-coordinate vector for $\vec{x}$.

$$\vec{b_1} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \vec{b_2} = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}, \vec{b_3} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \vec{c_1} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \vec{c_2} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \vec{c_3} = \begin{bmatrix} 0 \\ -1 \\ -2 \end{bmatrix}, [\vec{x}]_B = \begin{bmatrix} 1 \\ 0 \\ -9 \end{bmatrix}$$

$$P_B = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & -1 \\ 3 & 8 & 3 \end{bmatrix} \quad P_C = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & -1 \\ 4 & 5 & -2 \end{bmatrix}$$

$$P_B [\vec{x}]_B = P_C [\vec{x}]_C$$

$$[\vec{x}_B] = \underbrace{P_B^{-1} P_C}_{\underset{B \leftarrow C}{P}} [\vec{x}]_C$$

$$[\vec{x}]_C = \underbrace{P_C^{-1} P_B}_{\underset{C \leftarrow B}{P}} [\vec{x}]_B$$

$$\underset{B \leftarrow C}{P} = \begin{bmatrix} 3/2 & 1 & 1/2 \\ -1 & 1 & -1 \\ -5/2 & -2 & 3/2 \end{bmatrix} \quad \underset{C \leftarrow B}{P} = \begin{bmatrix} 1/4 & 3/4 & 3/4 \\ 1/2 & -1/2 & -1/2 \\ 1/4 & -11/4 & -5/4 \end{bmatrix}$$

$$[\vec{x}]_C = \begin{bmatrix} -13/2 \\ 5 \\ 23/2 \end{bmatrix}$$

4. For the vectors $\vec{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, *and* $\vec{v} = \begin{bmatrix} -6 \\ 9 \end{bmatrix}$, find the following:

a. $||\vec{u}||$

$\sqrt{4+9} = \sqrt{13}$

b. $\vec{u} \cdot \vec{v}$

$2(-6) + 3(9) = -12 + 27 = 15$

c. Are $\vec{u}$ *and* $\vec{v}$ orthogonal?

no, since the dot product is not $\vec{0}$