MTH 266, Quiz #11, Spring 2019 Name KEY

Instructions: You must show all work to receive full credit for the problems below. You may check your work with a calculator, but answers without work will receive minimal credit. Use exact answers unless the problem starts with decimals or you are specifically asked to round.

1. Define an inner product on the vector space of functions by $f \cdot g = \int_{-1}^{1} f(t)g(t)dt$. Determine if the functions $p(t) = 1 - t + t^2$, and $q(t) = -1 - t^2 + t^3$ are orthogonal.

$$\int_{-1}^{1} (1-t+t^2)(-1-t^2+t^3)dt =$$

$$\int_{-1}^{1} \cancel{t^5} - 2t^4 + \cancel{2t^3} - 2t^2 + \cancel{t} - 1 \, dt \qquad \int_{-a}^{a} \text{odd} = 0$$

$$= \int_{-1}^{1} -2t^4 - 2t^2 - 1 \, dt = 2\int_{0}^{1} -2t^4 - 2t^2 - 1 \, dt =$$

$$2\left[-\tfrac{2}{5}t^5 - \tfrac{2}{3}t^3 - t\right|_0^1 = 2\left[-\tfrac{2}{5} - \tfrac{2}{3} - 1\right] = -\frac{62}{15}$$

they are not orthogonal since the inner product is not 0.

2. Find an orthogonal basis for R^3 if one of the vectors is $\vec{u} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$.

answers will vary

$$\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = 2 - 2 + 0 = 0$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1/5 \\ 0 & 1 & -2/5 \end{bmatrix} \qquad \begin{aligned} x_1 &= -\tfrac{1}{5}x_3 \\ x_2 &= \tfrac{2}{5}x_3 \\ x_3 &= x_3 \end{aligned} \qquad \vec{x} = \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix}$$

Orthogonal basis $\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix} \right\}$

$$\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix} = -1 - 4 + 5 = 0$$

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$$\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix} = -2 + 2 + 0 = 0$$

3. Find the least squares line $y = \beta_0 + \beta_1 x$ that best fits the data $\{(2,3), (3,2), (5,1), (6,0)\}$.

$$\begin{array}{cc} \beta_0 & \beta_1 \\ \end{array}$$
$$\begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 5 \\ 1 & 6 \end{bmatrix} \vec{x} = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix} \qquad (A^T A)^{-1} A^T \vec{b} = \vec{x} = \begin{bmatrix} 4.3 \\ -0.7 \end{bmatrix} \begin{array}{c} \beta_0 \\ \beta_1 \end{array}$$

$$y = 4.3 - 0.7x$$

4. Determine if the vector $\vec{v} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$ is an eigenvector of the matrix $A = \begin{bmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix}$.

$$\begin{bmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 10+8+2 \\ 8+10+2 \\ 4+4+2 \end{bmatrix} = \begin{bmatrix} 20 \\ 20 \\ 10 \end{bmatrix} = 10 \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

Yes it is an eigenvector of A.