MTH 266, Quiz #2, Spring 2019 Name KEY

Instructions: You must show all work to receive full credit for the problems below. You may check your work with a calculator, but answers without work will receive minimal credit. Use exact answers unless the problem starts with decimals or you are specifically asked to round.

1. Consider the matrix $A = \begin{bmatrix} 1 & -2 & 7 & 5 \\ 0 & 3 & 0 & 3 \\ 1 & 2 & 0 & 2 \\ 1 & 5 & -1 & 4 \end{bmatrix}$. Do the vectors represented by the columns of the matrix span R^4? Why or why not? If they do, choose a random vector and prove it is a linear combination of the other vectors in the matrix and the multiples of each vector needed to obtain it. If they do not span R^4, find one vector outside the span and show that the system is inconsistent.

no. the reduced form is $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. it does not have 4 pivots and so cannot span $\mathbb{R}^4$ since vectors are not independent.

Consider, the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ augmenting (adding) to matrix

reduces to $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$ the system is inconsistent and therefore vector is not in the span

2. Determine if each of the sets below are linearly independent. Explain your reasoning in each case.

a. $\left\{\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ 5 \end{bmatrix}\right\}$ 2 vectors, not multiples of each other, so independent

b. $\left\{\begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 7 \\ 1 \end{bmatrix}\right\}$ more than 2 vectors in $\mathbb{R}^2$, so dependent

c. $\left\{\begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 5 \\ 1 \end{bmatrix}\right\}$ 2 vectors, not multiples So independent

d. $\left\{\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}\right\}$ reduces to $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ dependent no pivot in column 3

$v_3 = 2\vec{v}_1 + v_2$

e. $\left\{\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right\}$ dependent, 4 vectors in $\mathbb{R}^3$ are always dependent