

The exam will cover topics in Set Theory and Logic. Topics to review include:

- Set notation including $\cup, \cap, \emptyset, \{ \}, \in, \subset, A^c, -, |A|$
- Definitions such as union, intersection, empty set, mutually exclusive, subset, complement, universal set
- Translate set builder notation into listed set notation (interpreting the set elements)
- Be able to draw (shade) two- and three-set Venn Diagrams
- Identify a set notation expression that is equivalent to a two- or three-set shaded Venn diagram
- Solve application problems using Venn Diagrams
- Logic notation including $\vee, \wedge, \sim, \rightarrow, \leftrightarrow$, exclusive or
- Be able to construct truth tables
- Determine the validity of an argument
- Prove identities
- Use Logic Gates to determine the truth or falsehood based on inputs and convert to logical notation
- Take sentences in English and rewrite them in logical notation and rewrite logical notation as an English sentence.

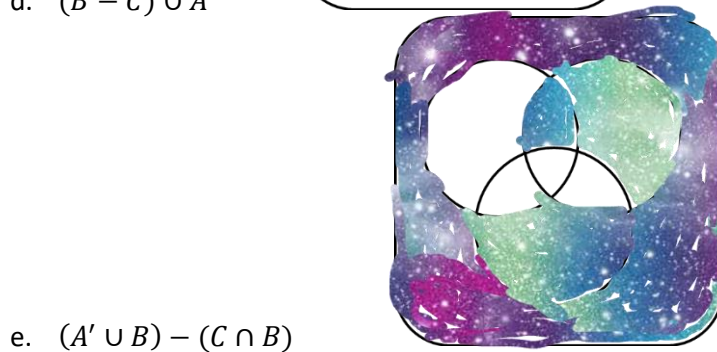
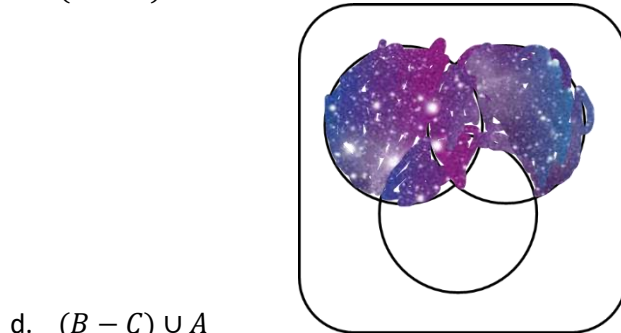
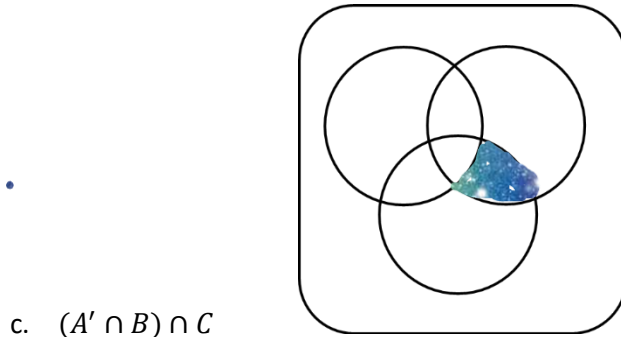
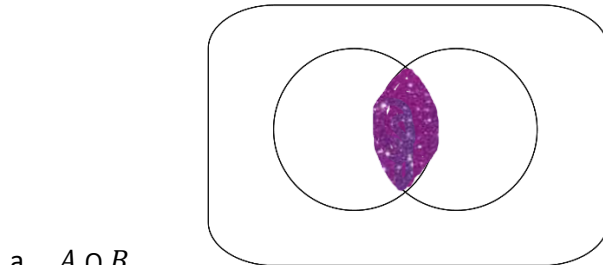
Practice Exam questions:

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each problem.

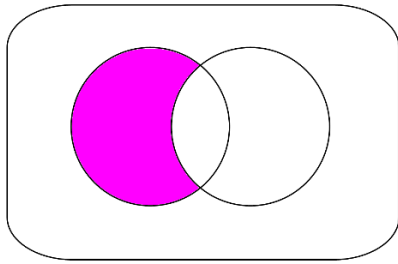
1. Let A be the set of letters in the name CAROLINGIAN and let B be set of letters in the name PERPENDICULAR.
 - a. List the elements in set A using proper set notation.
 $\{C, A, R, O, L, I, N, G\}$
 - b. List the elements in set B using proper set notation.
 $\{P, E, R, N, D, I, C, U, L, A\}$
 - c. Find $A \cap B$.
 $\{A, R, I, C, N, L\}$
 - d. Find $A \cup B$.
 $\{A, C, D, E, G, I, L, N, O, P, R, U\}$
 - e. What is the number of elements in set A, i.e. $n(A) = |A|$?
8
 - f. What is $|A \cup B|$?
12
 - g. What is $A' = A^c$?
 $\{B, D, E, F, H, J, K, M, P, Q, S, T, U, V, W, X, Y, Z\}$
2. Answer the following questions about sets:
 - a. List the elements in the set $C = \{x|x \text{ is an even counting number less than } 10\}$.
 $\{2, 4, 6, 8\}$
 - b. List the elements in set $D = \{x|x \text{ is an integer between } -1 \text{ and } 1 \text{ inclusive}\}$.
 $\{-1, 0, 1\}$
 - c. For each of the following questions, answer TRUE or FALSE.
 - i. $4 \in C$ TRUE
 - ii. $C \cap D = \emptyset$ TRUE
 - iii. $\emptyset \in C$ FALSE

iv. $1 \subset D$ FALSE if $\{1\} \subset D$ then TRUE

3. Draw a Venn Diagram that illustrates each of the following sets.

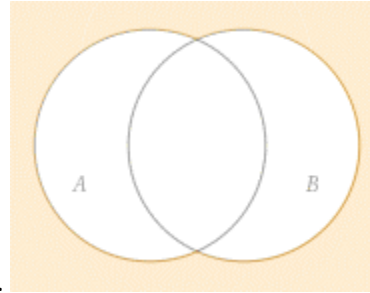


4. For each of the following Venn diagrams, write set notation that describes the indicated set.



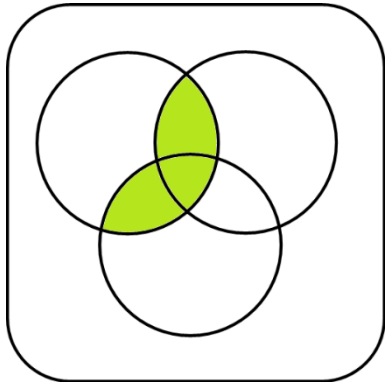
a.

$$A - B$$



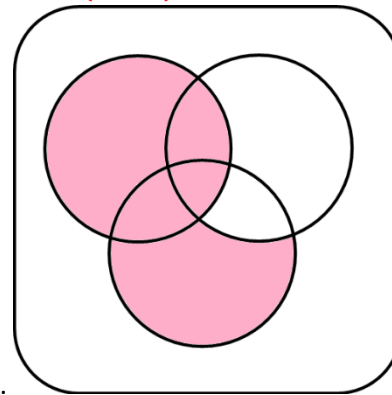
c.

$$(A \cup B)' = A' \cap B'$$



b.

$$(A \cap B) \cup (A \cap C)$$



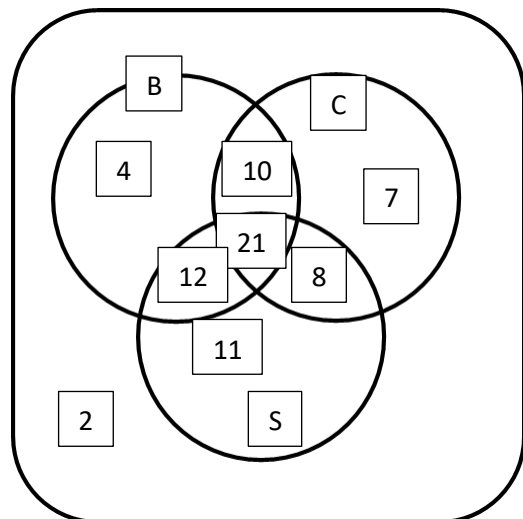
d.

$$(C - B) \cup A$$

5. A survey was conducted among 75 patients admitted to a hospital cardiac unit during a two-week period. Let B be the set of patients with high blood pressure, C be the set of patients with high cholesterol levels, and S the set of patients that smoke cigarettes. Fill in the Venn diagram below using the following data, and then use the diagram to answer the questions that follow.

- The number of patients with high blood pressure was 47
- The number of patients with high cholesterol was 46
- The number of patients who smoke is 52.
- The number of patients who smoke and have high blood pressure is 33
- The number of patients who both have high blood pressure and high cholesterol is 31
- The number of patients who have all three conditions is 21
- The number of patients with at least two conditions is 51

- a. Find the number of patients who had either high blood pressure or high cholesterol, but not both.
11
- b. Find the number of patients who had one or none of these conditions.
24
- c. Find the number of patients who have none of these conditions.
2



6. Let p be the statement “She has green eyes,” and let q be the statement “Andrew is 91 years old,” and r be the statement “The cat is lonely.” Use this information to translate the following symbolic statements into English sentences.

- a. $p \wedge q$ She has green eyes and Andrew is 91 years old.
- b. $\sim p \rightarrow q$ If she doesn't have green eyes, then Andrew is 91 years old.
- c. $(p \vee \sim q) \leftrightarrow r$ Only if she has green eyes or Andrew is not 91 years old, then the cat is lonely.

7. Construct truth tables for each of the following statements.

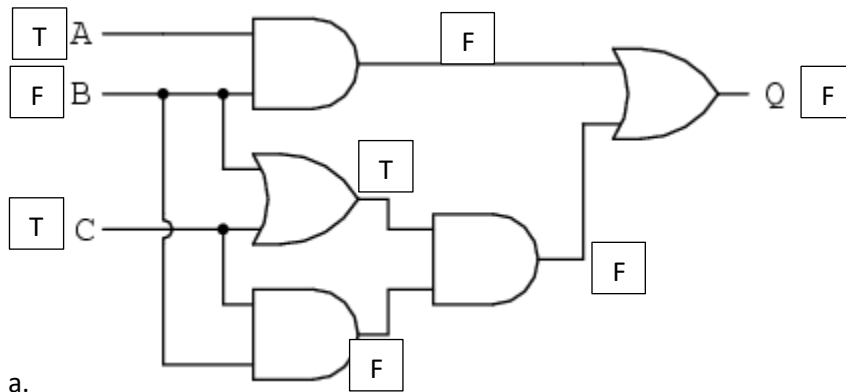
a. $p \wedge \sim q$

p	q	$\sim q$	$p \wedge \sim q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

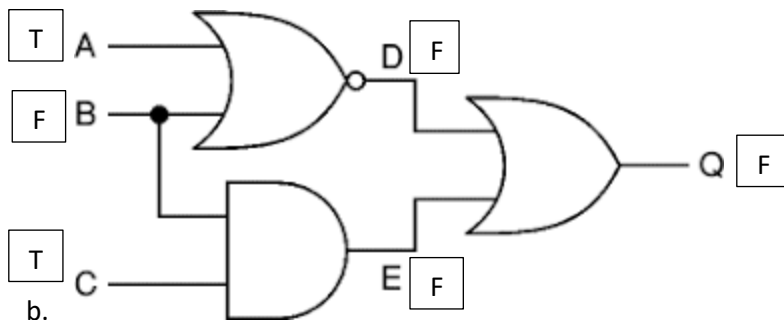
b. $(p \rightarrow q) \vee \sim r$

p	q	r	$p \rightarrow q$	$\sim r$	$(p \rightarrow q) \vee \sim r$
T	T	T	T	F	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	T	T	T
F	F	T	T	F	T
F	F	F	T	T	T

- 8. Explain in your own words the difference between “inclusive or” and “exclusive or”.
Inclusive or allows both to be true (true), but exclusive or is false if both are true.
- 9. Find the truth value of the logic gates below using the fact that A is True, B is False, and C is True.



a.



b.

10. Use determine the validity of the following argument. If the argument is invalid, explain why.

A mathematician is a device for turning coffee into theorems.

You turn coffee into theorems.

You are a mathematician.

Invalid. This structure does not guarantee you are a mathematician. You could be another kind of device that turns coffee into theorems. (It would be valid if you switched the last two statements.)