MT 112, Homework #3, Spring   Name ______________________________________

Instructions: Work problems on a separate sheet of paper and attach work to this page. You should show all work to receive full credit for problems. Checking your work with computer algebra systems is fine, but that doesn't count as "work" since you won't be able to use CAS programs on exams or quizzes. Sketch any graphs you obtain. Questions with compact answers can be recorded directly on this page. Graphs and longer answers that won't fit here, indicate which page of the work the answer can be found on and be sure to clearly indicate it on the attached pages.

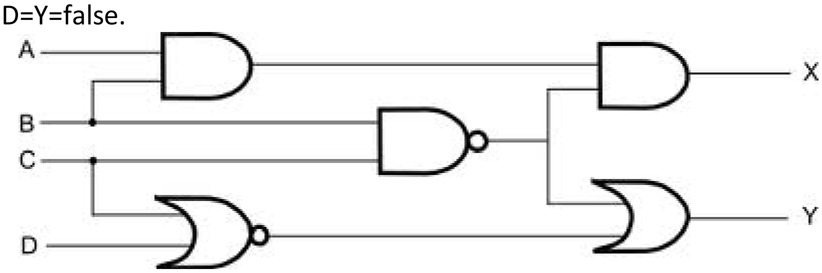
1. Let p and q be the propositions p: "I bought a lottery ticket", and q: "I won the million-dollar jackpot." Write each of the following compound propositions as English sentences.
   a. $\sim p$
   b. $p \to q$
   c. $p \leftrightarrow q$
   d. $\sim p \wedge \sim q$
   e. $p \vee q$
   f. $p \wedge q$
   g. $\sim p \to \sim q$
   h. $\sim p \vee (p \wedge q)$

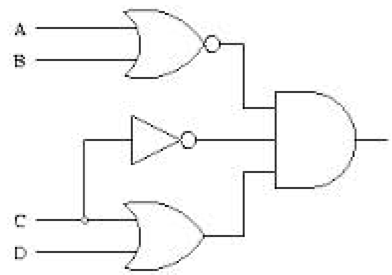
2. Let p and q be the propositions p: "You drive over 65 miles per hour", and q: "You get a speeding ticket." Write each of the following statement in terms of p, q, and logical connectives.
   a. You do not drive over 65 miles per hour.
   b. You drive over 65 miles per hour, but you do not get a speeding ticket.
   c. You will get a speeding ticket, if you drive over 65 miles per hour.
   d. If you do not drive over 65 miles per hour, you will not get a speeding ticket.
   e. Driving over 65 miles per hour is sufficient for getting a ticket.
   f. You get speeding ticket, but you do not drive over 65 miles per hour.
   g. Whenever you get a speeding ticket, you are driving over 65 miles per hour.

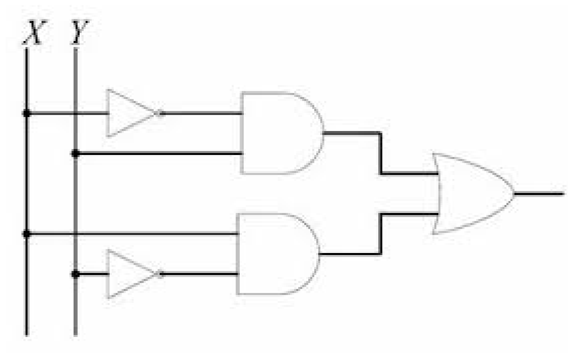
3. How many rows appear in the truth table for each of these compound propositions.
   a. $p \to \sim p$
   b. $q \vee p \vee \sim s \vee \sim r \vee \sim t \vee u$
   c. $(p \vee \sim r) \wedge (q \vee \sim s)$
   d. $(p \wedge r \wedge t) \leftrightarrow (q \wedge t)$

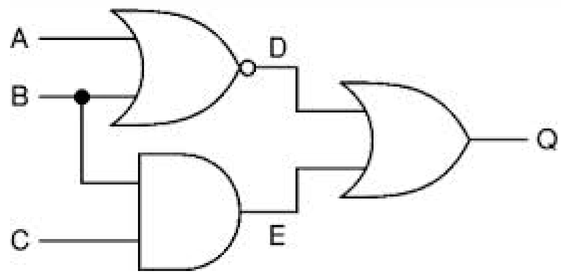
4. Construct a truth table for each of the following propositions.
   a. $p \to \sim p$
   b. $(p \leftrightarrow \sim q) \wedge (p \underline{\vee} q)$
   c. $(p \wedge q) \underline{\vee} (r \to s)$
   d. $p \underline{\vee} (p \vee q)$
   e. $(p \vee q) \wedge \sim r$

5. Find the output of each of the following logic gates/combinatorial circuits if A=C=X=true, and B=D=Y=false.

a. 

b. 

c. 

d. 

6. Use De Morgan's Laws to find the negation of "James is young and strong."

7. Proofs.
   a. Show that the statement $(p \wedge q) \rightarrow (p \rightarrow q)$ is a tautology.
   b. Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent.
   c. Show that the negation of a tautology is unsatisfiable. (You may do this with a specific tautology.)

8. What rule(s) of inference is being used in each of these arguments?
   a. Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major.
   b. Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major.
   c. Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.
   d. If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

9. For each of these collections of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

a. If I take the day off, it either rains or snows. I took Tuesday off or I took Thursday off. It was sunny on Tuesday. It did not snow on Thursday.
b. I am either clever or lucky. I am not lucky. If I am lucky, then I will win the lottery.
c. All rodents gnaw their food. Mice are rodents. Rabbits do not gnaw their food. Bats are not rodents.
d. All foods that are healthy to eat do not taste good. Tofu is healthy to eat. You only eat what tastes good. You do not eat tofu. Cheeseburgers are not healthy to eat.

10. Determine which of the following arguments are valid. If they are valid, what rules of inference are used? If it is not, what logical error occurs?
    a. If n is a real number such that $n > 1$, then $n^2 > 1$. Suppose that $n^2 > 1$. Then, $n > 1$.
    b. If n is a real number with $n > 2$, then $n^2 > 4$. Suppose that $n \leq 2$. Then $n^2 < 4$.
    c. If x is a positive real number, then x^2 is a positive real number. Therefore, if a^2 is a positive real number, where a is a real number, then a is a positive real number.