

143 Exam #1 Review Solutions

(1)

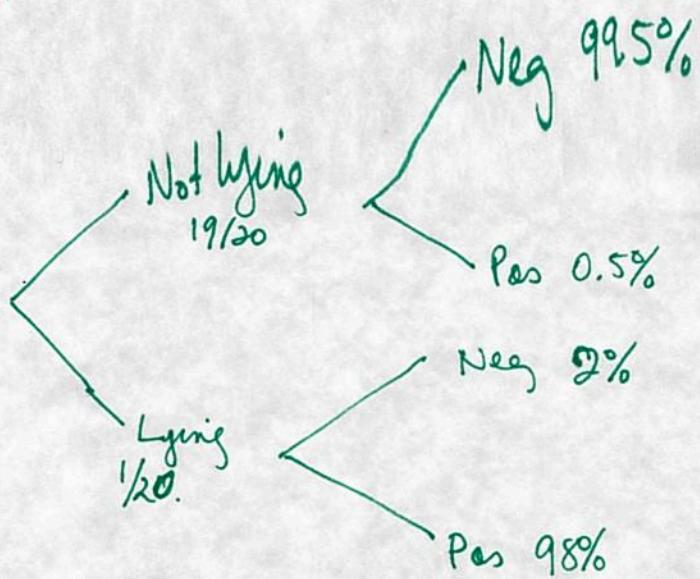
- #1 see attached table

2. 4% of the population is at or below the given value

3. 99.7%      4. 25<sup>th</sup> percentile

5. Classical is based on equally likely outcomes in the sample space  
Experimental probabilities are the result of observing a fixed # of events

6.



$$P(\text{Lying} | \text{Pos}) = \frac{P(\text{Lying} \cap \text{Pos})}{P(\text{Pos})} = \frac{\frac{1}{20} \cdot 0.98}{\frac{1}{20} \cdot 0.98 + \frac{19}{20} \cdot 0.005}$$

↑  
add up all cases that result  
in a positive test

$$= 0.9116279$$

or about 91%.

7.  $n = 50, p = 0.03$

a.  $x=0 \quad \binom{50}{0} (0.03)^0 (0.97)^{50} = \text{binompdf}(50, 0.03, 0) = 0.218065\dots$   
 $21.8\%$

b.  $x=1 \quad P(x \leq 1)$

$$= P(x=0) + P(x=1) = 21.8\% + \binom{50}{1} (0.03)^1 (0.97)^{49} =$$
 $\text{binomcdf}(50, 0.03, 1) = .5552\dots$

55.5%

$\dots + (50, 0.03, 4)$

## Major topics:

- Definitions
- Sampling Methods
- Frequency Distributions
- Making and Interpreting Graphs
- Calculating mean, median and mode, standard deviation, 5-number summary, percentiles, z-score
- Counting methods
- Probability rules
- Mean/expected value and standard deviation of discrete distributions
- Binomial probability distribution

## Sample problems.

1. Classify the following variables:

| Variable               | Categorical | Quantitative | Discrete | Continuous | Nominal | Ordinal | Interval | Ratio |
|------------------------|-------------|--------------|----------|------------|---------|---------|----------|-------|
| Test Grade<br>(Letter) | ✓           |              |          |            |         | ✓       |          |       |
| Continent              | ✓           |              |          |            | ✓       |         |          |       |
| Month                  | ✓           | ✓            | ✓        |            |         | ✓       | ✓        |       |
| Volume<br>(Sound)      |             | ✓            |          | ✓          |         |         | ✓        |       |
| Cost (\$)              |             | ✓            |          | ✓          |         |         |          | ✓     |
| Vegetable              | ✓           |              |          |            | ✓       |         |          |       |
| Goal                   | ✓           |              |          |            | ✓       |         |          |       |
| Altitude               |             | ✓            |          | ✓          |         |         |          | ✓     |
| Growth<br>Rate         |             | ✓            |          | ✓          |         |         |          | ✓     |
| Sexual<br>Orientation  | ✓           |              |          |            | ✓       |         |          |       |
| Year of<br>Birth       |             | ✓            | ✓        |            |         |         | ✓        |       |

2. What does it mean if we say a value represents the 4<sup>th</sup> percentile?
3. With symmetric/bell-shaped distributions, approximately what percent of the observations are within three standard deviations of the mean?
4. Expressed in percentiles, what does the first quartile represent?
5. Explain the difference between a classical (theoretical) probability and an experimental (observational) probability.
6. A particular model of lie detector test has a 98% probability of correctly detecting someone who is lying, and 99.5% probability of correctly detecting someone who is not lying. A human police officer interviews suspects using the lie detector, hoping to catch the 1 of the 20 suspects worth investigating further who they believe to be lying. If someone tests positively on the lie detector

**Stem-and-leaf of Problem9 N = 60**

5 5 05668  
15 6 3345567899  
27 7 122224566899  
(18) 8 011123344556777899  
15 9 00112456689  
4 10 0159

Leaf Unit = 1

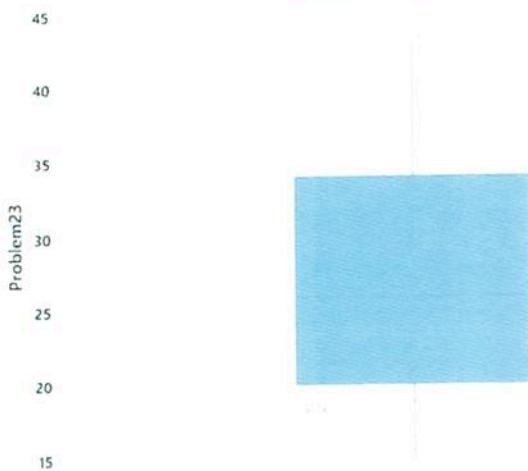
**Stem-and-leaf of Problem9 N = 60**

1 5 0  
5 5 5668  
8 6 334  
15 6 5567899  
21 7 122224  
27 7 566899  
(9) 8 011123344  
24 8 556777899  
15 9 001124  
9 9 56689  
4 10 01  
2 10 59

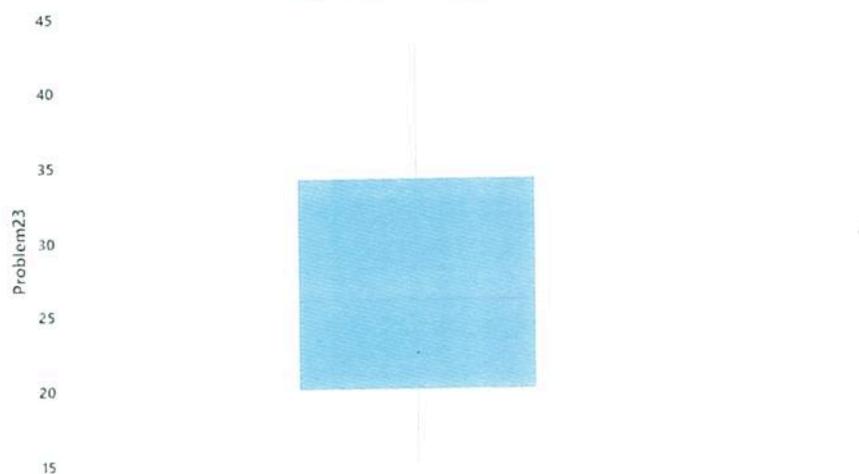
Leaf Unit = 1

**Statistics**

| Variable     | Total       |       |       |          |         |       |        |       |         |       |       |
|--------------|-------------|-------|-------|----------|---------|-------|--------|-------|---------|-------|-------|
|              | Count       | Mean  | StDev | Variance | Minimum | Q1    | Median | Q3    | Maximum | Range | IQR   |
| Problem9     | 60          | 79.98 | 13.44 | 180.66   | 50.00   | 69.50 | 81.00  | 89.75 | 109.00  | 59.00 | 20.25 |
| <b>N for</b> |             |       |       |          |         |       |        |       |         |       |       |
| Variable     | <b>Mode</b> |       |       |          |         |       |        |       |         |       |       |
| Problem9     | 72 4        |       |       |          |         |       |        |       |         |       |       |

**Boxplot of Problem23**

Boxplot of Problem23



**Statistics**

| Variable  | Total |                |       |          |         |       |        |       |         |       |
|-----------|-------|----------------|-------|----------|---------|-------|--------|-------|---------|-------|
|           | Count | Mean           | StDev | Variance | Minimum | Q1    | Median | Q3    | Maximum | Range |
| Problem23 | 15    | 26.87          | 9.10  | 82.84    | 15.00   | 20.00 | 26.00  | 34.00 | 43.00   | 28.00 |
| N for     |       |                |       |          |         |       |        |       |         |       |
| Variable  | IQR   | Mode           | Mode  |          |         |       |        |       |         |       |
| Problem23 | 14.00 | 15, 23, 26, 43 | 2     |          |         |       |        |       |         |       |

## 143 Exam #1 Review Key

9. See minitab data attached

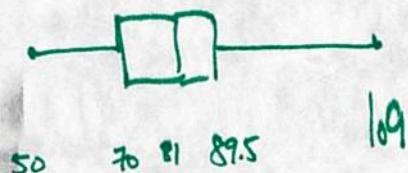
a.  $\rightarrow$  2 versions by 10's, by 5's

b.  $\bar{x} = 79.98$  median = 81 mode = 72

c.  $s = 13.44$  range =  $109 - 50 = 59$

d. min = 50,  $Q_1 = 70$ ,  $Q_2 = \text{med} = 81$ ,  $Q_3 = 89.5$ , max = 109

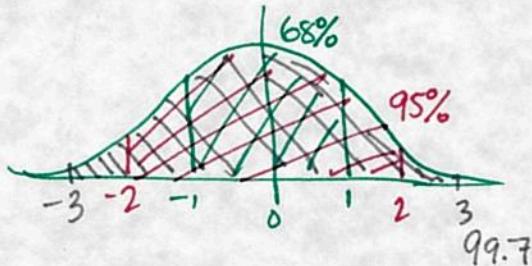
e. See minitab graph attached.



f.  $\frac{17}{60} \rightarrow \frac{20}{60}$

28<sup>th</sup> + 33<sup>rd</sup>

10.



11. a.  $\frac{88 - 72}{5} = 3.2$

b. 99.9? roughly since it is well above 3

c.  $67 \sim -1\sigma$  and  $82 \sim +2\sigma$   $\frac{68 + 99.7}{2} = 81.5$

12. a. roughly symmetric or slightly right skewed

b. right skewed

c. none

d. roughly uniform

13. a.  $6/100$  b.  $42/100$  c.  $1 - 42/100 = 58/100$  d.  $\frac{2}{100} \cdot \frac{8}{99} \cdot \frac{1}{98} = 1.649 \times 10^{-5}$

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## 143 Exam #1 Review Key

156.  $\binom{42}{6} = 5,245,786$

c.  $10 \cdot 9 \cdot 8 = 720$

d.  $\frac{12!}{2! \cdot 2!} = 119,750,400$

16.  $a. \left(\frac{1}{2}\right)^9 = \frac{1}{512} = 0.00195$

b.  $\frac{13 \cdot \left(\frac{4}{3}\right)\left(\frac{48}{2}\right)}{\left(\frac{52}{5}\right)} = 0.022569 \approx 2.3\%$

17a. 126 b. 210 c. 792 d. 1

18.

|   |   |    |    |    |    |    |    |    |
|---|---|----|----|----|----|----|----|----|
| * | 1 | 2  | 3  | 4  | 5  | 6  | 7  | 8  |
| 1 | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 2 | 3 | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
| 3 | 4 | 5  | 6  | 7  | 8  | 9  | 10 | 11 |
| 4 | 5 | 6  | 7  | 8  | 9  | 10 | 11 | 12 |
| 5 | 6 | 7  | 8  | 9  | 10 | 11 | 12 | 13 |
| 6 | 7 | 8  | 9  | 10 | 11 | 12 | 13 | 14 |
| 7 | 8 | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |

$$\frac{3}{64}$$

19.  $0(0.07) + 1(0.13) + 2(0.18) + 3(0.30) + 4(0.22) + 5(0.08) + 6(0.02)$

$$= 2.79$$

|      |       |       |       |         |
|------|-------|-------|-------|---------|
| x    | 490   | 90    | 40    | -10     |
| P(x) | 1/450 | 1/450 | 2/450 | 446/450 |

21.  $n=10, p = \frac{2}{3}$

a.  $\binom{10}{8} \left(\frac{2}{3}\right)^8 \left(\frac{1}{3}\right)^2 = \text{binompdf}(10, \frac{2}{3}, 8) = 0.19509\dots$   
 $19.5\%$

b.  $P(X \leq 8) = \text{binomcdf}(10, \frac{2}{3}, 8) = 0.89595 \approx 89.6\%$

c.  $\mu = E(X) = np = 10 \left(\frac{2}{3}\right) = 6.67$

22. a.  $\frac{1640 - 1498}{199} = 0.7135\dots$

b.  $\frac{28 - 21}{5.2} = 1.346\dots \quad \frac{1910 - 1498}{199} = 2.07\dots$

Barbara did better

23. See attached minitab work

a.  $26.87 = \text{mean } \bar{x}$ , median = 26, mode = none

b. st. dev = 9.10 range = 28

c. min = 15,  $Q_1 = 20$ , Med = 26,  $Q_3 = 34$ , Max = 43