1/16/2021

Set Notation

Set: what is it?
A set is a list of elements (unordered)
An example: $S = \{B, E, T, S, Y\}$

$$T = \{B, E, S, T, Y\}$$

If all the elements of two sets are the same, then the sets are equal. $S = T$

Universal sets: the set of all possible elements for a given situation

For the S set, the universal set might be the set that contains all 26 letters of the English alphabet.

Set builder notation

$$\{a\ variable\ |conditions\}$$

$$\{x|x > 10, x\ is\ an\ integer\}$$

"x such that x is an integer greater than 10"

$$\{11, 12, 13, 14, 15, \ldots\}$$

$$\{x|x\ is\ an\ integer, 3 \leq x \leq 7\}$$

Is equivalent to

$$\{3,4,5,6,7\}$$

$$\{x|x\ is\ a\ letter\ in\ my\ last\ name\}$$

Is equivalent to

$$\{M, C, A, L\}$$

Do not list repeated elements in the set more than once.

Sets are the groups; elements are the objects in the set

Mathematical notation for asking whether or not an element is contained in a set.

ϵ = "is an element of"

$$A = \{3,4,5,6,7\}$$

Is $4 \in A$?
"is 4 an element of the set A"?
Is $8 \in A$? False; $8 \notin A$: "8 is not an element of the set A"

$$A = \{2, 4, 6, 8\}$$

$$B = \{3, 6, 9\}$$

$$C = \{4,8\}$$

Subset notation: $\subset$ "is a subset of"
A subset of another set is a set where all the elements of the first set are contains in the second set

$$C \subset A$$
$$B \not\subset A$$

B is not a subset of A because there is at least one element of B that is not also in A

Bad notation: $6 \subset B$ bad because 6 is not a set; 6 is an element of B, which is written $6 \in B$
$\{6\} \subset B$ is okay because {6} is a set

The empty set: $\{\quad\}, \emptyset$
The empty set is a set, but it contains no elements.

$$D = \{cats, dogs, birds, fish, gerbils, lizards, snakes, hamsters \ldots\}$$

The set of all pets I currently have: $E = \{\quad\}$
Use subset notation with the empty set, because it's a set

The empty set is considered to be a subset of every set

What are all the subsets of C?
$\subset vs. \subseteq$ there is no distinction

$$\{\quad\}, \{4\}, \{8\}, \{4,8\}$$

Subsets of B:

$$\{\quad\}, \{3\}, \{6\}, \{9\}, \{3,6\}, \{3,9\}, \{6,9\}, \{3,6,9\}$$

Three more relationships with sets:

Complement or negation

$$U = \{x | the\ capital\ letters\ of\ the\ English\ alphabet\}$$

$$S = \{B, E, T, S, Y\}$$

$S' = \bar{S} = \sim S$: the set of all elements that are in the universal set but that are **not** in the set S

$$S' = \{A, C, D, F, G, H, I, J, K, L, M, N, O, P, Q, R, U, V, W, X, Z\}$$

$$U = \{x | all\ positive\ integers\ less\ than\ 10\}$$

$$A = \{x | even\ positive\ integers\ less\ than\ 10\}$$

$$U = \{1,2,3,4,5,6,7,8,9\}$$
$$A = \{2,4,6,8\}$$

$$A' = \{1,3,5,7,9\}$$

Union and intersection
Union of two sets A, B, is the set of all elements contained in **either** set A **or** set B. ∪
Intersection of two sets A, B is the set of all elements that are contained in **both** A **and** B. ∩

$$A = \{2, 4, 6, 8\}$$

$$B = \{3, 6, 9\}$$
$$C = \{4,8\}$$

$$A \cup B = \{2,4,6,8,3,9\} = \{2,3,4,6,8,9\}$$
$$A \cap B = \{6\}$$

Order of operations: parentheses, negation, intersection, union

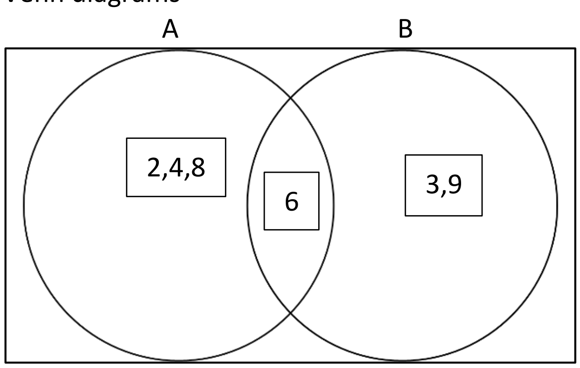
$$A' \cup B$$
The complement of A first, then union the result with B
$$(A \cap B)'$$
Do inside the parentheses first, and then take the complement of the result

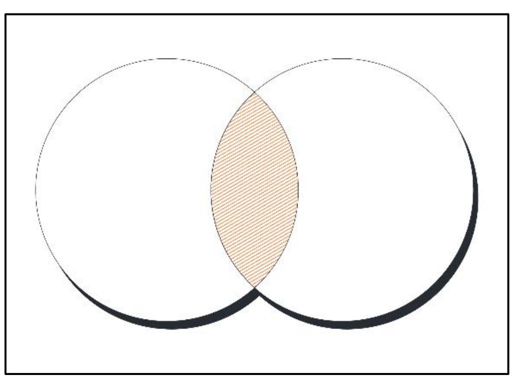
$$A - B$$
All the elements of A that are not also in B: $A \cap B'$ is equivalent

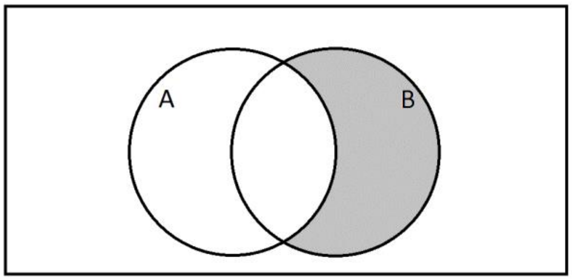
Cross product, cartesian product
$$B \times C$$
Creates a set of ordered pairs where the first element is in B, and the second element is from C
$$B \times C = \{(3,4), (3,8), (6,4), (6,8), (9,4), (9,8)\}$$

$$B \times C \times A = \{(3,4,2), (3,8,2), (6,4,2), (6,8,2), (9,4,2), (9,8,2), (3,4,4), (3,8,4), (6,4,4), (6,8,4), (9,4,4), (9,8,4), (3,4,6), (3,8,6), (6,4,6), (6,8,6), (9,4,6), (9,8,6), (3,4,8), (3,8,8), (6,4,8), (6,8,8), (9,4,8), (9,8,8)\}$$

Venn diagrams

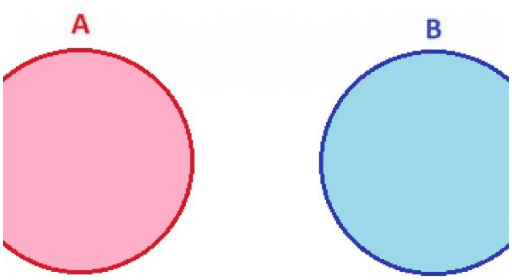



intersection of two sets

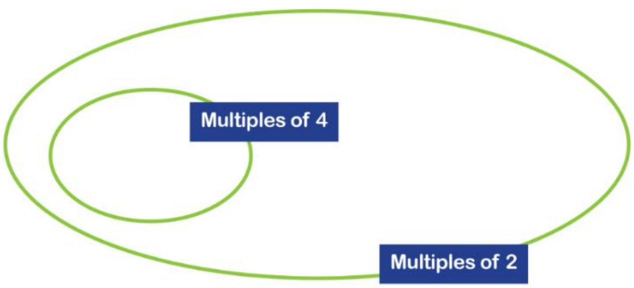


$B - A$

Mutually exclusive: have an empty intersection



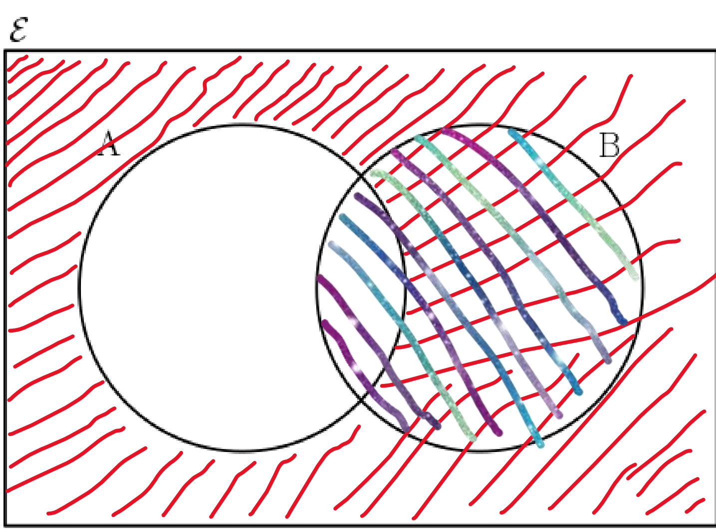
there is no intersection {}, because there is no overlap between the sets, have no elements



subset relation

$A = \{2,4,6,8,10,12 \ldots\}$

$B = \{4,8,12, \ldots\}$



$A' \cup B$

Any shading is the union. Any overlap is the intersection.

Logical notation

Logic and sets are closely related, but uses statements instead of sets.

Statement: sentence: The snow is falling. John is speaking.
The snow is falling = p
John is speaking = q

Negation: The snow is not falling. ~p

And: The snow is falling, and John is speaking: p ∧ q (but)
Or: The snow is falling or John is speaking: p ∨ q

If, then: If the snow is falling, then John is speaking: p → q (p implies q)
Iff (if and only if): p ↔ q (biconditional, equivalency)
Only if the snow is falling is John speaking. The snow is falling if and only if John is speaking.

"Backwards E" = ∃= there exists (at least one thing exists that will make the statement true)
"Upside A" = ∀= for all (this is always the case)

∀ x, x^2 >=0 (if x is real)
∃ x such that x^2=1.

~p∧q : The snow is not falling and John is speaking

If p is true, then ~p is false.
And: the combined statement is only true if both things are true.
Or: is inclusive rather than exclusive: is only false if both statements are false
If then: if p is true, and q is true, it's true; but if p is false, then q can be anything (only false when p is true and q is false)
Biconditional: both are true or both are false makes the whole statement true; if they don't match, then the combined statement is false.

n(A) = the number of elements in A = |A|

Z is the set of integers
N is the set natural numbers {1,2,3,...}
R is the set of real numbers