1/26/2021

(Alternate free textbook: Math in Society ~ covers several of the topics we are covering)

Set Notation

Set is a collection of elements ~ a box that holds things

The basic way to describe sets is through set notation:

$$\{list\ of\ elements\}$$

A is the set of all letters in my first name:

$$A = \{B, E, T, S, Y\} = \{B, E, S, T, Y\} = \{Y, B, S, E, T\}$$

B is the set of letters in my last name:

$$B = \{M, C, A, L\}$$

Sets do not contain repetition.
Sets are equal if all the elements in the first set are in the second set, and vice versa (in any order)

Set builder notation:

$$\{x|conditions\}$$

The set of all whole numbers that are less than 10.

$$\{x|x < 10, x\ is\ a\ whole\ number\}$$

$$\{0,1,2,3,4,5,6,7,8,9\}$$

$$\{0,1,2, \dots ,9\}$$

Even numbers: $\{2, 4,6,8, \dots \}$

Special case: the empty set:

$$\{\ \} = \emptyset$$

Notation for "being an element of" a set ∈

$$A = \{B, E, T, S, Y\}$$

Is $T \in A$?

This is true, yes, the element T is in the set A.

Is $Q \in A$? False. As a true statement: $Q \notin A$.

B is the set of all even numbers. Is this a true statement: $5 \in B$?

Subset: is a collection of objects, but all the objects in the subset are also elements of a larger set.

$$B = \{2, 4, 6, 8, 10, 12, \dots\}$$
$$C = \{4, 8, 12, 16, \dots\}$$
$$D = \{3,6,9,12,15, 18, \dots\}$$

Subset notation is $\subset$ or $\subset$

Is $C \subset B$? This is true: any number divisible by 4 is also even

$$D \not\subset B$$

All the elements of D are not also in B, so D is NOT a subset of B.

$$\subset or \subseteq$$

$$B = \{M, C, A, L\}$$

What are all the possible subsets:
$\emptyset, \{M\}, \{A\}, \{C\}, \{L\}, \{M, C\}, \{M, A\}, \{M, L\}, \{C, A\}, \{C, L\}, \{A, L\}, \{M, C, A\}, \{M, C, L\}, \{M, A, L\}, \{A, C, L\}, \{M, C, A, L\}$

The empty set is a subset of every set

Every element in the subset is also an element of the "larger" set

4 is an element, but {4} is a set of one element, whose element is 4

Relations between sets that are not subsets

Universal set: the set of all possible things that could be in a set

Ex. U = The universal set is the set of all letters in the English alphabet (capitals)

$$A = \{B, E, T, S, Y\}$$

Negative/complement of set A: compare set A to the universal set, and the negative of A is any element in the universal set, which is not in A.

$$\sim A, \bar{A}, A^c, A', etc.$$

$$A' = \{A, C, D, F, G, H, I, J, K, L, M, N, O, P, Q, R, U, V, W, X, Z\}$$

Union and Intersection

Union of sets $A \cup B$: the set of all elements that are in either A or B (the set contains all elements of A, or B, but no duplicates)

Intersection of sets $A \cap B$: the set of all elements that are in both A and B (only those elements that the sets have in common)

$$H = \{1, 4, 7, 10, 11, 14\}$$
$$J = \{2, 5, 7, 8, 11, 13, 15\}$$

What is $H \cup J$? $\{1,2,4,5,7,8, 10, 11, 13, 14, 15\}$
What is $H \cap J$? $\{7, 11\}$

$A - B$: the set of all elements in A, where any element of B has been removed

$$H - J: \{1, 4, 10, 14\}$$

Essentially A but remove the intersection with B.

$$J - H: \{2, 5, 8, 13, 15\}$$

Count the elements in the set:
$|A| = n(A)$: the number of elements in the set
Cardinality is a term for the number of elements in a set

$$|H| = 6$$

Venn diagrams

Drawings that usually involve circles to represent sets, and they are a visual way of thinking about set relations.

Mutually exclusive sets: the two sets don't overlap at all; they have an empty intersection = have nothing in common (disjoint)

Cross product ×

$$A = \{1, 2, 3\}$$
$$B = \{x, y\}$$

$$A \times B = \{(1, x), (1, y), (2, x), (2, y), (3, x), (3, y)\}$$

The number of elements in $A \times B$ is the number of elements in A times the number of elements in B

Logic

Statements are the main object being considered in logic, and we are going to connect them, or do operations on them similar to subsets, equality, or, and, negation.

p "proposition" = It is snowing.
q = Joe is standing by the road.

Negation $\sim p$: It is not snowing.
And (Conjunction) $p \wedge q$: It is snowing and Joe is standing by the road.
Or (Disjunction) $p \vee q$: It is snowing or Joe is standing by the road.
If, ... then (Conditional) $p \rightarrow q$: If it is snowing, then Joe is standing by the road.
If and only if (iff), (Biconditional), equality: $p \leftrightarrow q$: If it is snowing, then that happens only if Joe is standing by the road (and vice versa). "only happens when"

It is snowing but Joe is standing by the road.

$$p \wedge q$$

Joe is standing by the road, if it is snowing.

$$p \rightarrow q$$

Because, since

The number of solutions that satisfy a statement

∀ = "for all", "for every" – every possible value of a variable works

∃ = "there exists", "there is at least one" – there is at least one solution for this expression
! = "unique", there is exactly one solution

$$\forall x(|x| \geq 0), x \in R$$

R = is the set numbers, Z = is the set of integers (positive, negative or 0 whole numbers), N = counting numbers (positive whole numbers)

The statement is true for all x that is real, so the whole statement is true.

$$\exists x(x^2 = 3), x \in Z$$

This is statement is false because the solution is not an integer (not a whole number)

$$\exists!\, x(x^2 = 4), x \in Z$$

The solutions are 2, and -2… so it's not unique. Two solutions therefore false.

$$\exists!\, x(x^2 = 0), x \in Z$$

Then this is true because there is only one solution (0).