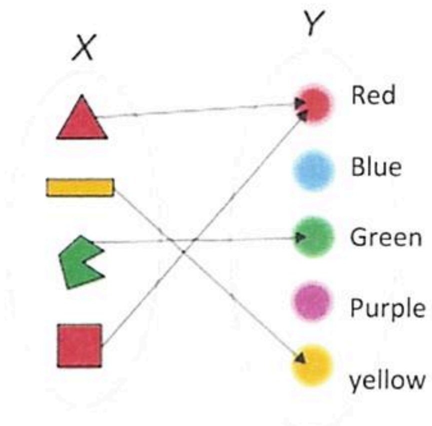
MTH 161, Exam #1, Spring 2021 Name KEY

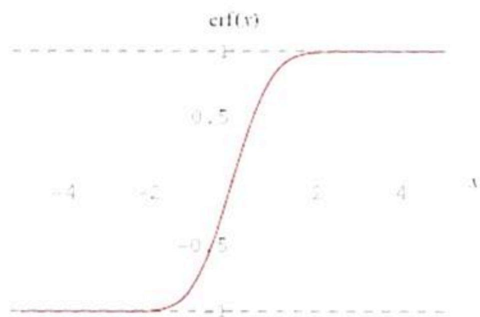
Instructions: Show all work. Give exact answers unless specifically asked to round. All complex numbers should be stated in standard form, and all complex fractions should be simplified. If you do not show work, problems will be graded as "all or nothing"; partial credit will not be possible.

1. For each of the following relations, determine i) the domain and range, ii) if the relation is a function, iii) if it is a function, is its inverse also a function. (6 points each)



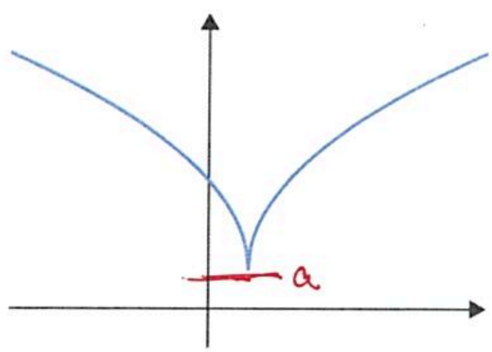
a.

D: {△, ▭, ᐸ, □}
R: {Red, blue, Green, purple, yellow}
is a function
the inverse is not a function



b.

D: $(-\infty, \infty)$
R: $(-1, 1)$
it is a function
it's inverse is a function



c.

D: $(-\infty, \infty)$
R: $[a, \infty)$
it is a function
it's inverse is not a function

2. Consider the points $(-2,-5),(3,-1)$. Find: (5 points each)
   a. The distance between the two points

$$d = \sqrt{(-2-3)^2 + (-5-(-1))^2} = \sqrt{25+16} = \sqrt{41}$$

$$\quad -5 \qquad\qquad -5+1$$
$$\qquad\qquad\qquad -4$$

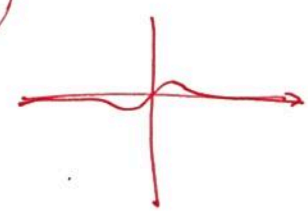
   b. The midpoint of the line segment connecting the points.

$$\left(\frac{-2+3}{2}, \frac{-5-1}{2}\right) = \left(\frac{1}{2}, -3\right)$$

3. For each of the following functions, determine:
   i) any intervals on which the function is increasing, $(-1,1)$
   ii) intervals on which the function is decreasing, $(-\infty,-1),(1,\infty)$
   iii) intervals on which the function is constant, no where
   iv) any relative extrema (relative maxima or minima), $(-1,-1/2)$ min, $(1,1/2)$ max
   v) symmetry (even, odd or neither). odd

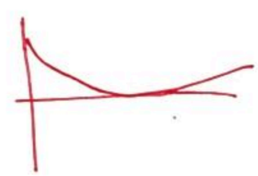
   [Hint: it's helpful to sketch the graph.] (10 points each)

   a. $f(x) = \frac{x}{x^2+1}$ for a



   b. $f(x) = \left|\sqrt{x+5} - 11\right|$

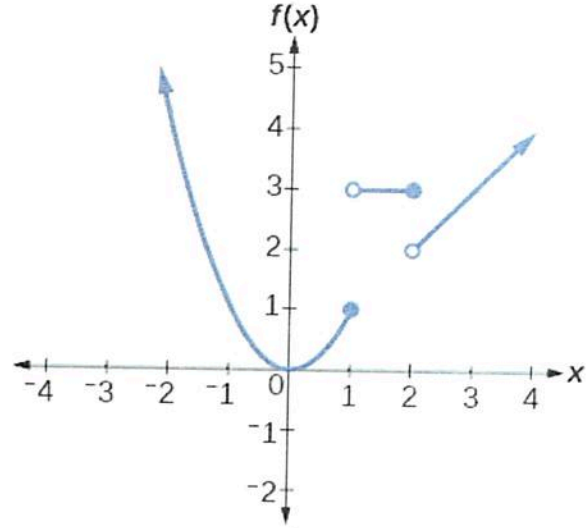
min $(116, 1.75 \times 10^{-7})$
max $(-5, 11)$



increasing $(116, \infty)$
decreasing $(-5, 116)$
not constant
no symmetry

4. Write an equation of the piecewise graph shown. (8 points)

$$f(x) = \begin{cases} x^2 & x \le 1 \\ 3 & 1 < x \le 2 \\ x & x > 2 \end{cases}$$



5. Find $\frac{f(x+h)-f(x)}{h}$ for $f(x) = -3x^2 + x - 1$. (10 points)

$x^2 + 2xh + h^2$

$$\frac{-3(x+h)^2 + (x+h) - 1 - (-3x^2 + x - 1)}{h} =$$

$$\frac{-\cancel{3x^2} - 6xh - 3h^2 + \cancel{x} + h \cancel{-1} + \cancel{3x^2} - \cancel{x} \cancel{+1}}{h} =$$

$$\frac{-6xh - 3h^2 + h}{h} = \frac{\cancel{h}(-6x - 3h + 1)}{\cancel{h}} = -6x - 3h + 1$$

6. What is the average rate of change of the function $f(x) = 1 - x^3$ on the interval $[-1,2]$? (6 points)

$f(-1) = 1 - (-1)^3 = 1 - (-1) = 2$ $(-1, 2)$

$f(2) = 1 - (2)^3 = 1 - 8 = -7$ $(2, -7)$

$$m = \frac{-7 - 2}{2 - (-1)} = \frac{-9}{3} = \boxed{-3}$$

7. Find an equation of the line with the following properties: (5 points each)
   a. Passing through the points $(2,5)$ and $(6,-1)$.

$$m = \frac{5-(-1)}{\cancel{2}-6} = \frac{6}{4} = \frac{3}{2}$$

$$y+1 = \frac{3}{2}(x-6) \to$$
$$y+1 = \frac{3}{2}x - 9 \to y = \frac{3}{2}x - 10$$

   b. Perpendicular to the line $3x+4y=12$ and passing through $(1,5)$.

$$4y = -3x+12$$
$$y = -\frac{3}{4}x + 3 \qquad \perp \text{ slope is } \frac{4}{3}$$

$$y-5 = \frac{4}{3}(x-1) \to$$
$$y-5 = \frac{4}{3}x - \frac{4}{3} \to y = \frac{4}{3}x + 11/3$$

   c. Parallel to $y=7$ and passing through $(2,-3)$.

$$y = -3$$

8. If $f(x) = |x|$, write the function that has the following transformations applied in the given order: (8 points)
   a. Shift left 9 units
   b. Reflect over the x-axis
   c. Compress by a factor of 3
   d. Shift down by 2

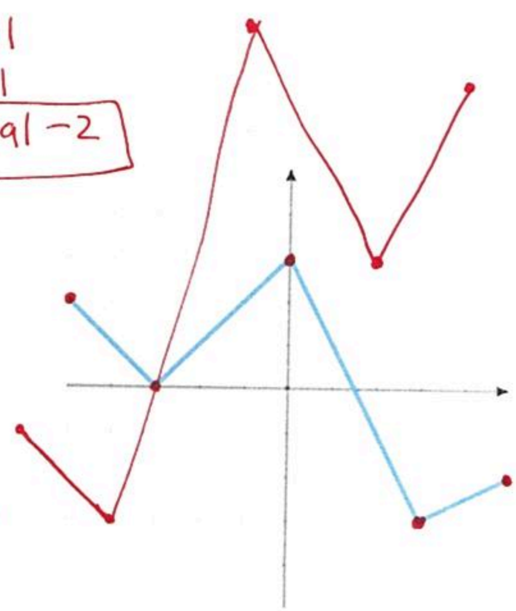
$g(x) = |x+9|$
$h(x) = -|x+9|$
$j(x) = -\frac{1}{3}|x+9|$
$\boxed{k(x) = -\frac{1}{3}|x+9| - 2}$

9. Shown is the function $f(x)$. Sketch the graph of $2f(-x+1)+3$. (10 points)

2 stretch → reflect over x

$2f[-(x-1)]+3$ shift +1

$(-5,2) \to (-4,2) \to (4,2) \to (4,4) \to (4,7)$
$(-3,0) \to (-2,0) \to (2,0) \to (2,0) \to (2,3)$
$(0,3) \to (1,3) \to (-1,3) \to (-1,6) \to (-1,9)$
$(3,-3) \to (4,-3) \to (-4,-3) \to (-4,-6) \to (-4,-3)$
$(5,-2) \to (6,-2) \to (-6,-2) \to (-6,-4) \to (-6,-1)$



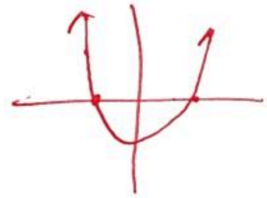
10. Create a sign chart to solve the quadratic inequality $2x^2 - x - 3 > 0$. Write the solution in interval notation. (5 points)

$(2x-3)(x+1) > 0$

$x = 3/2 \quad x = -1$

$+ \quad - \quad +$

$-1 \quad 3/2$



$(-\infty, -1) \cup (3/2, \infty)$

11. Create a linear regression equation for the data shown below. Write the equation. Interpret the slope in the context of the problem. (10 points)

	A	B
1	Age (x)	Average Amount Spent on Medical Expenses (per month in Rs) (y)
2	15	100
3	20	135
4	25	135
5	37	150
6	40	250
7	45	270
8	48	290
9	50	360
10	55	375
11	61	400
12	64	500
13	67	1000
14	70	1500
15		

$y = 16.89x - 355.32$

for every additional year of age, one can expect to spend an additional \$16.89

12. Given $f(x) = x^2 + 1, g(x) = \sqrt{x-4}, h(x) = x + \frac{1}{x}$, find the following functions and state the domain. (5 points each)

a. $(g+h)(x)$

$= \sqrt{x-4} + x + \frac{1}{x}$ domain $[4, \infty)$

$x \geq 4$ $x \neq 0$

b. $\left(\frac{f}{h}\right)(x)$ $= \dfrac{x^2+1}{x+\frac{1}{\cancel{x}}} \cdot \dfrac{x}{x} = \dfrac{x^3+x}{x^2+1}$ domain $(-\infty, 0) \cup (0, \infty)$

$x \neq 0$

c. $(g \circ f)(x)$ $= \sqrt{x^2+1-4} = \sqrt{x^2-3}$ domain

$x^2 - 3 \geq 0$

$x^2 \geq 3$

$x \leq -\sqrt{3}, x \geq \sqrt{3}$

$(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

$x \geq 4$

13. Find the inverse of $f(x) = \frac{x+1}{x-2}$. Sketch the graph and its inverse on the same graph. Describe the symmetry you see. (10 points)

$y = \dfrac{x+1}{x-2}$ $(0, -\frac{1}{2})$ $(1, 0)$

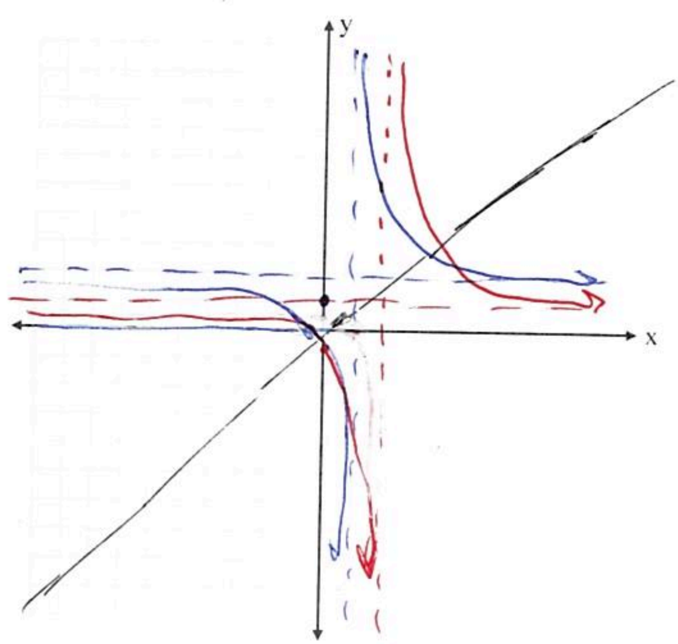
$x = \dfrac{y+1}{y+2}$

$xy + 2x = y + 1$

$xy - y = -2x + 1$

$y(x-1) = -2x + 1$

$y = \dfrac{-2x+1}{x-1}$ $(-\frac{1}{2}, 0)$ $(0, 1)$



Symmetry w/ line $y = x$

14. The endpoints of a circle's diameter are $(-3,-4)$ and $(6,-8)$. Find the center of the circle, its radius, and equation in standard form. (8 points)

$$\text{Center } \left(\frac{-3+6}{2}, \frac{-4-8}{2}\right) = \left(\frac{3}{2}, -6\right)$$

$$\text{radius} = \frac{1}{2}\text{ diameter} = \frac{1}{2}\sqrt{(\underbrace{-3-6}_{-9})^2 + (\underbrace{-4+8}_{4})^2} =$$

$$\frac{1}{2}\sqrt{81+16} = \frac{1}{2}\sqrt{97} = \frac{\sqrt{97}}{2}$$

$$\left(x-\frac{3}{2}\right)^2 + (y+6)^2 = \frac{97}{4}$$

15. Let $P(x,y)$ be a point on the graph of $y = x^2 - 8$. Express the distance d from P to the point $(2,4)$, as a function of the point's x-coordinate. Find the minimum distance graphically. (6 points)

$$d = \sqrt{(2-x)^2 + (\underbrace{4-x^2+8}_{12-x^2})^2} =$$

$$\sqrt{4-4x+x^2+144-24x^2+x^4} =$$

$$\sqrt{148-4x-23x^2+x^4}$$

abs. min $\approx$ $(3.43, 1.45)$

16. Find the vertex, the y-intercept and any zeros (real and complex) of the function $f(x) = -2(x+1)^2 - 5$. Use that information to sketch the graph. [If the zeros are complex, you may need to plot additional points by hand; if so, use the symmetry of the graph.] (8 points)

vertex $(-1,-5)$

$-2(1)^2-5=-7$ $\quad (0,-7)$

$(-2,-7)$

$-2(\underbrace{2)^2}_{4}-5=-13$ $\quad (1,-13)$

$(-3,-13)$

no real zeros

Symmetry axis $\quad x=-1$