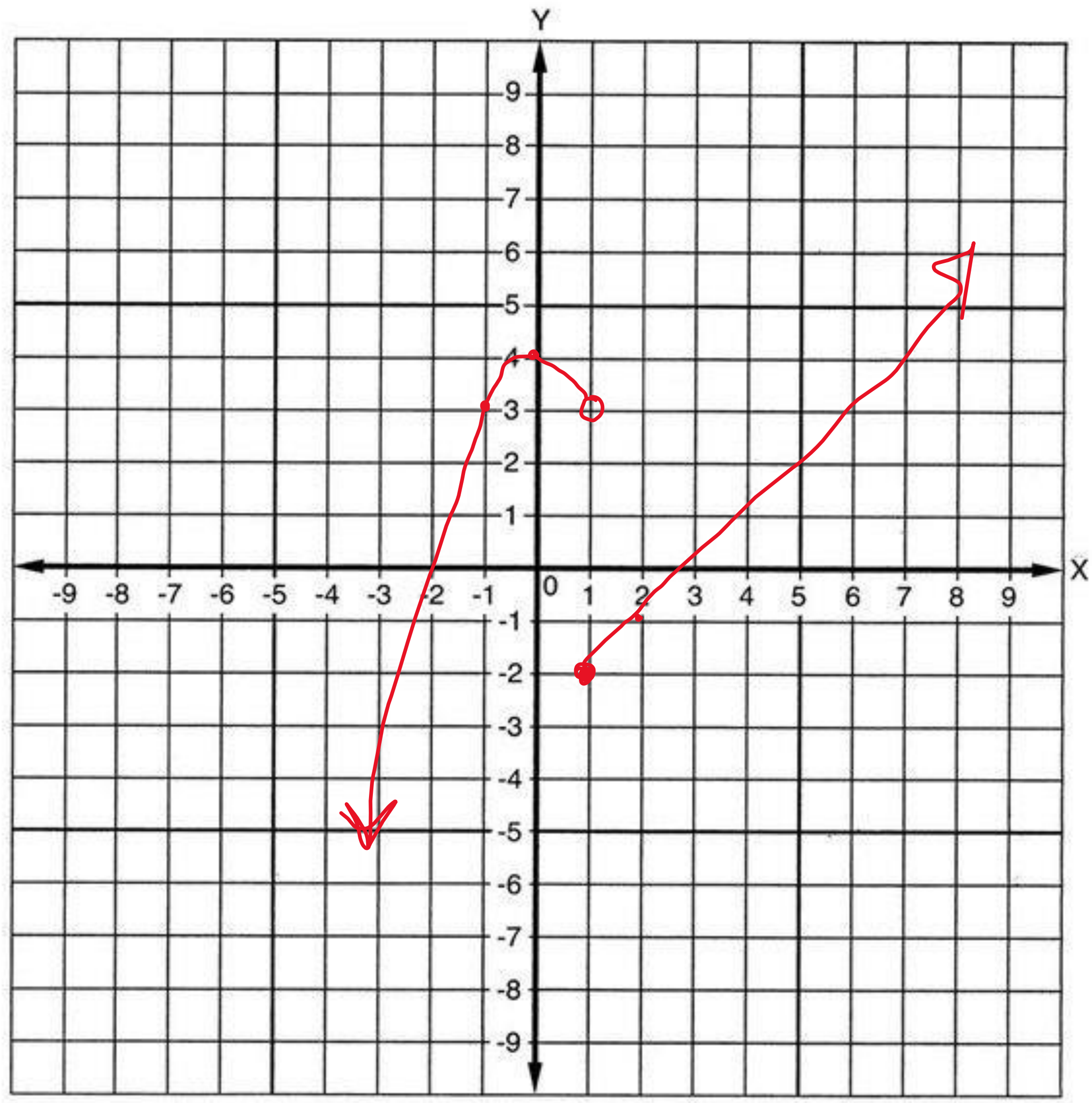
2/4/2021

$$f(x) = \begin{cases} 4 - x^2, & x < 1 \\ x - 3, & x \geq 1 \end{cases}$$

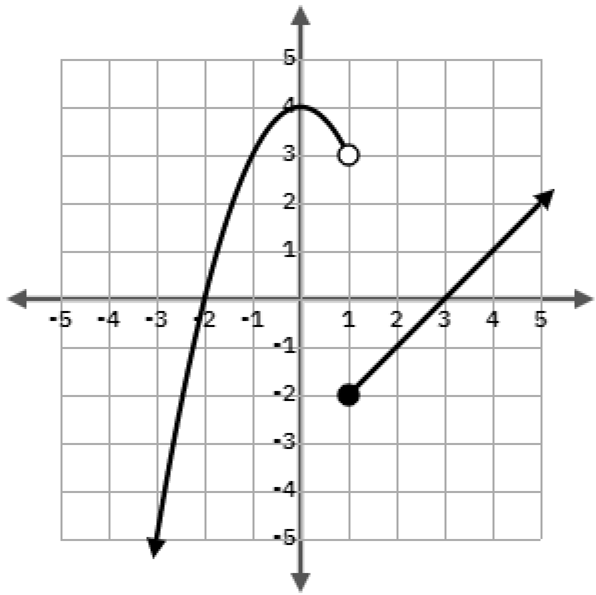
1) For each piece of the graph, we need to plot the point where the function piece ends (where the inequality tells us that the function breaks), and then at least one other point on each piece.
2) You mark the endpoints with open circles where the inequality is $<$ or $>$. And closed circles where there is a $\geq or \leq$.



First piece, plot open circle at x=1. $4 - x^2 \rightarrow 4 - (1)^2 = 3$ (1,3)
Then for x=0, $4 - (0)^2 = 4$, (0,4)
For x= -1, $4 — (-1)^2 = 3, (-1,3)$

2nd piece: plot at closed circle at x=1: $x-3 \rightarrow 1-3 = -2$, (1,-2)
X=2: 2-3 = -1, (2,-1)

Increasing $(-\infty, 0) \cup (1, \infty)$
Decreasing $(0,1)$
Domain $(-\infty, \infty)$
Range $(-\infty, \infty)$
Min: local min of -2 at x=1, no absolute min
Max: local max of 4 at x=0, no absolute max
No symmetry

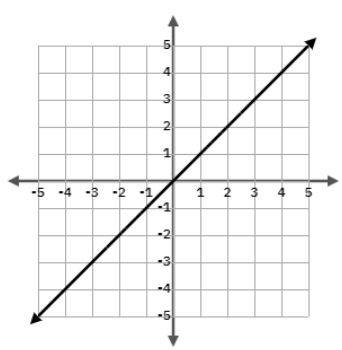


Piecewise graphs can have more than two pieces.

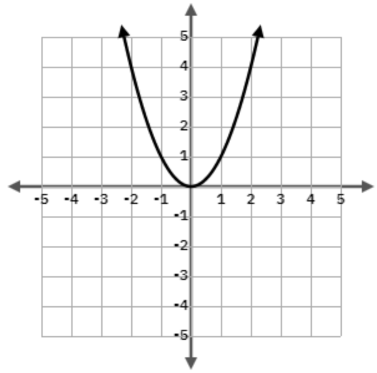
$$f(x) = \begin{cases} x-1, & x < -3 \\ x^2, & -3 \le x < 2 \\ -x+4, & x \ge 2 \end{cases}$$

Transformations

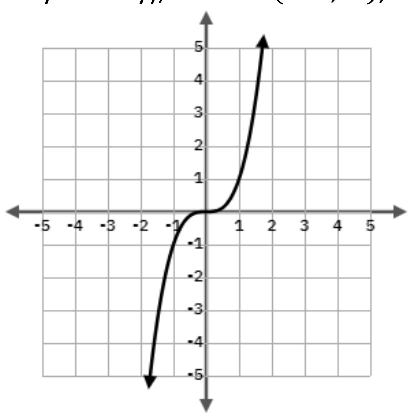
Is a method of graphing complex equations based on base graphs.



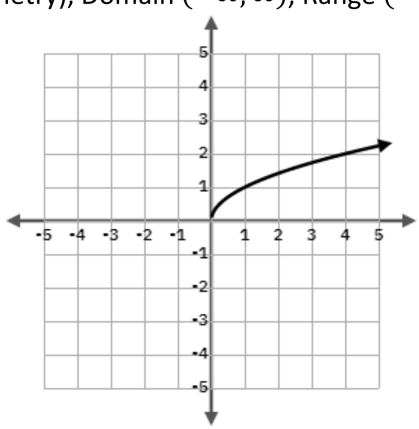
Identity Graph/Linear Function $y = x$
Domain $(-\infty, \infty)$, Range $(-\infty, \infty)$, Odd symmetry (origin symmetry)



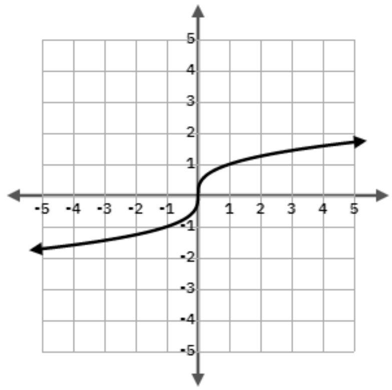
Quadratic/Square graph $y = x^2$
Even symmetry (y-axis symmetry), Domain $(-\infty, \infty)$, Range $[0, \infty)$



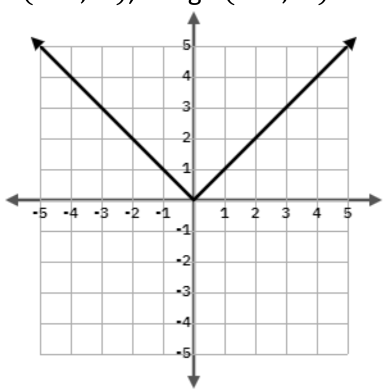
Cubic Graph $y = x^3$
Odd symmetry (origin symmetry), Domain $(-\infty, \infty)$, Range $(-\infty, \infty)$



Square Root Graph $y = \sqrt{x}$
No symmetry, Domain $[0, \infty)$, Range $[0, \infty)$



Cube Root Graph $y = \sqrt[3]{x}$
Odd symmetry (origin), Domain $(-\infty, \infty)$, Range $(-\infty, \infty)$



Absolute Value Graph $y = |x|$ $f(x) = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$

Even symmetry (y-axis), Domain $(-\infty, \infty)$, Range $[0, \infty)$

Horizontal Shift – move the base graph either left or right h units

$$f(x - h)$$

Shift $y = x^2$ three units to the right (+3), then function becomes $f(x) = (x - 3)^2$

Shift $y = |x|$ two units to the left (-2), then the function becomes $f(x) = |x + 2|$

Horizontal shifts going INSIDE the operation of the function: replacing x with x-h (note the sign change in equation)

Vertical Shift – move the base graph up or down k units

$$f(x) + k$$

Shift $y = x^2$ three units UP, then the function becomes $f(x) = x^2 + 3$

Shift $y = |x|$ two units DOWN, then function becomes $f(x) = |x| - 2$

Vertical direction keeps the sign and apply outside the function operation.

Reflections

Horizontal Reflection (across the y-axis), replace x with -x

$$f(-x)$$

We want to reflect $y = x^3$ across the y-axis (horizontally), then the function becomes $y = (-x)^3$
Or reflect $y = \sqrt{x}$ across the y-axis, then function becomes $y = \sqrt{-x}$.

Vertical reflections

Vertical reflection (across the x-axis), tack on a – (minus) sign to the whole function

$$-f(x)$$

If we want to reflect $y = x^3$ across the x-axis (vertically), then the function becomes $y = -x^3$
$y = \sqrt{x}$ across the x-axis: $y = -\sqrt{x}$

If want to reflect $y = x^2$ across the x-axis, then the function becomes $y = -x^2$

Stretching/Compression

Horizontal stretching/compressing, multiply x by ax (multiply by some factor a)

$$f(ax)$$

If a<1 stretch in the x-direction, a>1 is compression in the x-direction

Vertical stretching/compressing, multiply the function by a

$$af(x)$$

If a>1, stretch in the y-direction, and if a<1, compress in the y-direction

Applying transformation has to be done in a particular

1) Apply the horizontal reflection (or stretch/compression) (multiply)
2) Apply the horizontal shift (add/subtract)
3) Apply the vertical reflection (or stretch/compression) (multiply)
4) Apply the vertical shift (add/subtract)

Apply the following transformations to a graph $y = x^2$:
1) Shift left four $y = (x+4)^2$
2) Reflect across the x-axis $y = -(x+4)^2$
3) Stretch vertically by ½ $y = -\frac{1}{2}(x+4)^2$

Average Rate of Change (Difference Quotient, Slope)

$$\frac{\Delta f(x)}{\Delta x} = \frac{f(b) - f(a)}{b - a} = \frac{y_2 - y_1}{x_2 - x_1} = m = \frac{f(x+h) - f(x)}{h} = \frac{f(x+h) - f(x)}{x + h - x}$$

$$x + h = b, x = a$$

$$f(x) = x^2 - 10x + 45$$

What is the average rate of change from $x = 1$ to $x = 3$? What is the average rate of change on the interval $[1,3]$?

$$f(1) = 1 - 10 + 45 = 36$$
$$f(3) = 9 - 30 + 45 = 24$$

$$(1,36), (3,24)$$

$$\frac{24 - 36}{3 - 1} = -\frac{12}{2} = -6$$

-6 means that for each unit change in x, the average rate of change is that y decreases by 6

Solving absolute value equations.

$$2|5x + 1| - 3 = 0$$

$$2|5x + 1| = 3$$

$$|5x + 1| = \frac{3}{2}$$

$5x + 1 = -\frac{3}{2}$	or	$5x + 1 = \frac{3}{2}$
$5x = -\frac{5}{2}$	or	$5x = \frac{1}{2}$
$x = -\frac{1}{2}$	or	$x = \frac{1}{10}$

$$|x - 3| = |x + 2|$$

$$x - 3 = x + 2$$
$$-(x - 3) = -(x + 2)$$
$$-3 = 2 \text{ No Solution}$$

$$-(x - 3) = x + 2$$
$$(x - 3) = -(x + 2)$$
$$-x + 3 = x + 2$$
$$-2x = -1$$
$$x = \frac{1}{2}$$

Quadratic Functions

General form $y = ax^2 + bx + c$
Standard (vertex) $y = a(x-h)^2 + k$
(h,k) is the vertex

Converting from general to standard:
Method 1: complete the square.

$$y = x^2 - 2x + 3$$

$$x^2 + 2hx + h^2 = (x+h)^2$$

If there is a coefficient of x^2 factor it out.
Look at coefficient of x: -2. This is equal to 2h. Solve for h.

$$-2 = 2h$$
$$h = -1$$
$$(x^2 - 2x + 1) + 2$$
$$y = (x-1)^2 + 2$$

Method 2: find the vertex, and then use the coefficient of x^2

$$x_{vertex} = -\frac{b}{2a}$$

$$x = \frac{2}{2 \times 1}, x = h = 1$$

Plug this value into the original equation to obtain k (y-coordinate of the vertex)

$$k = y = (1)^2 - 2(1) + 3 = 2$$

$$(h,k) = (1,2)$$

$$y = 1(x-1)^2 + 2$$

Axis of symmetry: vertical line that goes through the vertex.

$$x = h$$

Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant $b^2 - 4ac$
Is negative: no real solutions, 2 complex solutions
Is zero: one real solutions
Is positive: two real solutions

Inequalities of functions.

$$x > x^2 - 4$$

$$x^2 - x - 4 < 0$$

Transformations (examples)

$$y = 5f(2x + 1) + 3$$

There is a base function $f(x)$.

Vertical shift up of 3
Vertical stretch of 5
Horizontal compression of $\frac{1}{2}$
Horizontal shift left of ½

$$f(2x + 1) = f\left[2\left(x + \frac{1}{2}\right)\right]$$

$$g(x) = \sqrt{2(x - 3)} + 1$$

$$y = \sqrt{x}$$

$\sqrt{2x}$ h. compression of ½
$\sqrt{2(x-3)}$ h. shift of 3 to the right
$\sqrt{2(x-3)} + 1$ v. shift up 1

Base graph $y = \sqrt{x}$	(0,0)	(1,1)	(4,2)	(9,3)
h. Compression of ½	(0,0)	(1/2,1)	(2,2)	(9/2,3)
h. shift right of 3	(3,0)	(7/2,1)	(5,2)	(15/2,3)
v. shift up 1	(3,1)	(7/2,2)	(5,3)	(15/2,4)