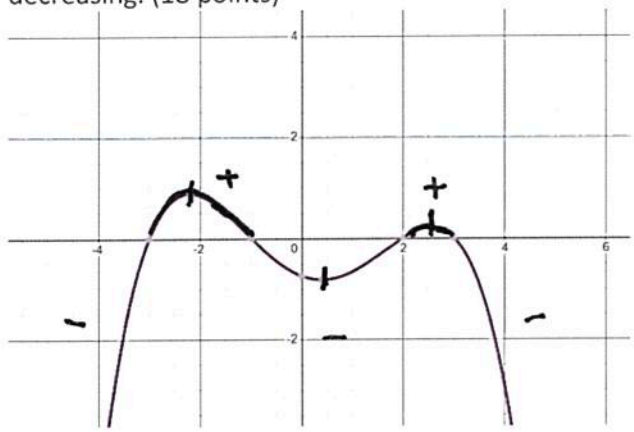
MTH 263, Exam #2, Spring 2022 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

Part 1: These questions you will submit answers to in Canvas. Show all work and submit the work with Part 2 of the exam. But you must submit the answers in Canvas to receive credit. Each question/answer will be listed separately. The Canvas question will refer to the number/part to indicate where you should submit which answer. The questions will appear in order (in case there is an inadvertent typo). Correct answers will receive full credit with or without work in this section, but if you don't submit work and clearly label your answers, you won't be able to challenge any scoring decisions for making an error or any kind.

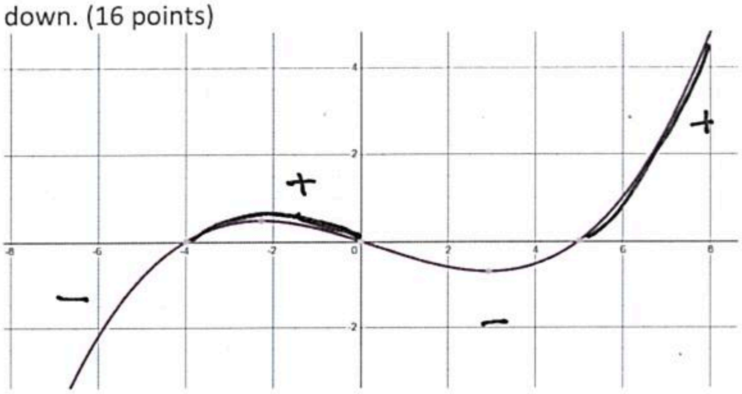
1. Given the graph of $f'(x)$ shown below, determine the intervals on which $f(x)$ is increasing or decreasing. (18 points)



increasing $(-3,-1) \cup (2,3)$

decreasing $(-\infty,-3) \cup (-1,2) \cup (3,\infty)$

2. Given the graph of $f''(x)$, determine the intervals on which $f(x)$ is concave up or concave down. (16 points)



Concave up $(-4,0) \cup (5,\infty)$

Concave down $(-\infty,-4) \cup (0,5)$

3. Use differentials to estimate the value of $\sqrt[3]{27.05}$. Round your answer to 6 decimal places if needed. (8 points)

(exact is 3.00185071)

$f(x) = \sqrt[3]{x}$, $x = 27$, $dx = 0.05$

$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$ $f'(27) = \frac{1}{3}\cdot\frac{1}{3^2} = \frac{1}{27}$

$f(27) = 3$ $f(27.05) \approx 3 + \frac{1}{27}(0.05) = 3.002$

3.001851852

4. Find the critical points of $f(x) = 3x^4 - 4x^3 - 12x^2 + 6$. (12 points)

$f'(x) = 12x^3 - 12x^2 - 24x = 12(x^3 - x^2 - 2x) = 12x(x^2 - x - 2)$

$12x(x-2)(x+1)$

$\boxed{x = 0, 2, -1}$

5. Find the absolute extrema of $f(x) = 3x^4 - 4x^3 - 12x^2 + 6$ on the interval [-3,3]. (12 points)

all critical points (from #4) in interval

$f(-3) = 249$ ← absolute max @ $x=-3$ is 249

$f(-1) = 1$

$f(0) = 6$

$f(2) = -26$ ← absolute min @ $x=2$ is -26

$f(3) = 33$

6. Find the value of each limit. (10 points each)

a. $\lim\limits_{x\to\infty} \cos\left(\frac{1}{x}\right) = 1$

b. $\lim\limits_{x\to\infty} (3x)^{1/x} = e^0 = 1$

$\lim\limits_{x\to\infty} \ln(3x)^{1/x} = \lim\limits_{x\to\infty} \frac{1}{x}\ln(3x) = \lim\limits_{x\to\infty} \frac{\ln(3x)}{x} \overset{\frac{\infty}{\infty}}{=} \lim\limits_{x\to\infty} \frac{\frac{1}{3x}\cdot 3}{1} = 0$

e

7. Use Newton's Method with the function $f(x) = 3x^2 - 11x + 3$ with an initial guess $x_0 = 1$ to find to root. What is the value of the iteration x_2? What is the value of the root after the 4th decimal place stops changing? [Round to 4 decimal places.] (20 points)

$x_2 = 0.272727$

$x_5 = 0.296743$ root

etc.

See Excel file

8. The position of the particle is defined by the equation $s(x) = 8x^3 - x^4$. At what point is the **velocity** a maximum? Round your answer to 4 decimal places if needed. (10 points)

$s'(x) = 24x^2 - 4x^3 = 4x^2(6-x)$ $x=0,\ x=6$ critical points

$s''(x) = 48x - 12x^2$

$12x(4-x)$

$x=6$ is a maximum

9. Use a Riemann sum and the right-hand rule to estimate the value of $\int_1^4 \sqrt{x} - \frac{1}{x}\,dx$ using 6 rectangles. After calculating your estimate, round to 4 decimal places. (15 points)

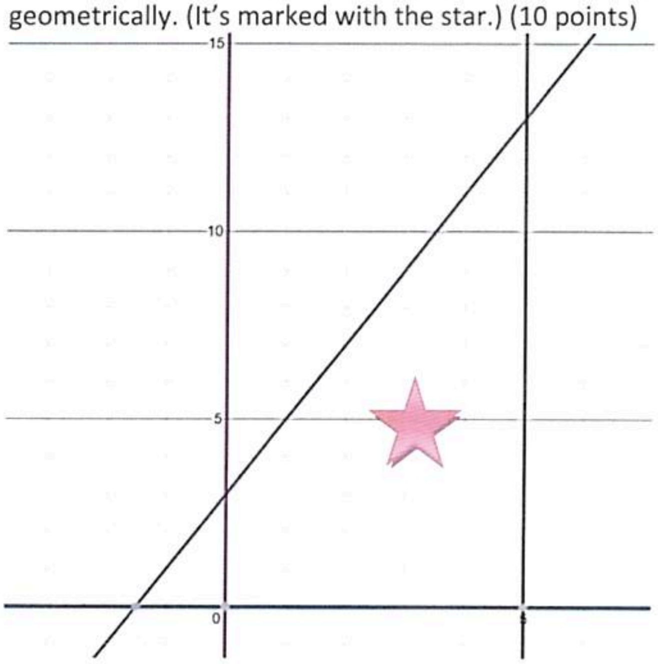
$\Delta x = \frac{4-1}{6} = \frac{3}{6} = \frac{1}{2}$

$\sum f(x_i)\Delta x = \frac{1}{2}\left[\sqrt{1.5} - \frac{1}{1.5} + \sqrt{2} - \frac{1}{2} + \sqrt{2.5} - \frac{1}{2.5} + \sqrt{3} - \frac{1}{3} + \sqrt{3.5} - \frac{1}{3.5} + \sqrt{4} - \frac{1}{4}\right]$

≈ 3.6936

(the true value is ≈ 3.28 so this is the correct ballpark)

10. Find the area of the region in the graph below bounded by $y = 2x + 3, y = 0, x = 0, x = 5$ geometrically. (It's marked with the star.) (10 points)



$A = \frac{1}{2}bh$ using triangles

$= \frac{1}{2}(6.5)(13) - \frac{1}{2}(1.5)(3)$

$42.25 - 2.25 = 40$

$A = \frac{1}{2}h(b_1 + b_2)$

$= \frac{1}{2}5(3+13) =$

$\frac{1}{2}\cdot 5 \cdot 16 = 5.8 = 40$

Part 2: In this section you will record your answers on paper along with your work. After scanning, submit them to a Canvas dropbox as directed. These questions will be graded by hand.

11. Create a sign chart for the first and second derivatives of $f(t) = 3t^4 - 8t^3 - 18t^2$. Identify and critical points and inflection points. Use this information to sketch the graph of the function. (20 points)

$f'(t) = 12t^3 - 24t^2 - 36t = 12t(t^2 - 2t - 3) \stackrel{12}{=} t(t-3)(t+1) = 0$

$t = 0, 3, -1$

$f''(t) = 36t^2 - 48t - 36 = 12(3t^2 - 4t - 3) = 12(3t$?

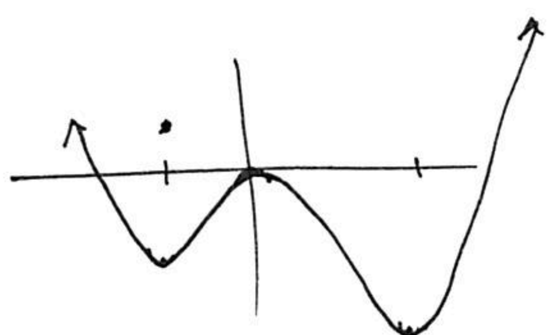
$t = \frac{4 \pm \sqrt{16+36}}{6} = \frac{4 \pm \sqrt{52}}{6} = \frac{4 \pm 2\sqrt{13}}{6} = \frac{2}{3} \pm \frac{\sqrt{13}}{3}$

$-$ $+$ $-$ $+$ $f'(t)$

-1 0 3

$+$ $-$ $+$ $f''(t)$

$2/3 - \sqrt{13}/3$ $2/3 + \sqrt{13}/3$

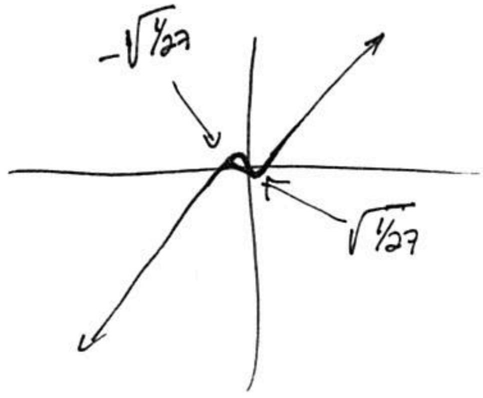


12. Create a sign chart for the first and second derivatives of $f(x) = x - \sqrt[3]{x}$. Identify any critical points, cusps, and inflection points. Use this information to sketch a graph of the curve. (20 points)

cusp
undefined at $x=0$

$f'(x) = 1 - \frac{1}{3}x^{-2/3} = 1 - \frac{1}{3\sqrt[3]{x^2}} = 0 \qquad 1 = \frac{1}{3\sqrt[3]{x^2}} \rightarrow \frac{1}{3} = \sqrt[3]{x^2} \rightarrow x = \pm\sqrt{\frac{1}{27}}$

$f''(x) = -\frac{1}{3}\cdot(-2/3)x^{-5/3} = \frac{-2}{9\sqrt[3]{x^5}} \neq 0$ undefined at $x=0$ never $=0$



vertical tangent at $x=0$

13. Find the antiderivative of $f(x) = x - \frac{1}{x^2}$. What is the value of the constant if $F(1) = 6$. (10 points)

$$\int x - \frac{1}{x^2}\,dx = \int x - x^{-2}\,dx = \frac{1}{2}x^2 + \frac{1}{x} + C$$

$$\frac{1}{2}(1)^2 + \frac{1}{1} + C = 6$$

$$\frac{3}{2} + C = 6 \rightarrow C = 9/2$$

14. Find the antiderivatives. (12 points each)

a. $\int x^e + 4\sin(2x) - 1dx$

$$\frac{1}{e+1}x^{e+1} - 2\cos(2x) - x + C$$

b. $\int \sec^2(x) - \csc x \cot x - \frac{1}{x\sqrt{x^2-1}}dx$

$$\tan x + \csc x - \operatorname{arcsec} x + C$$

c. $\int \cosh 9x - \frac{1}{x+4} - \pi^{-x}dx$

$$\frac{1}{9}\sinh 9x - \ln|x+4| + \frac{1}{\ln\pi}\pi^{-x} + C$$

d. $\int \frac{e^x}{1+e^{2x}}dx$

$$= \arctan(e^x) + C$$

15. Find the derivative of $g(x) = \int_1^{\ln x} 4^t + \cos(t)\,dt$. [i.e. find $\frac{dg}{dx}$.] (10 points)

$$4^{\ln x} \cdot \frac{1}{x} + \cos(\ln x) \cdot \frac{1}{x}$$

$$\frac{4^{\ln x}}{x} + \frac{\cos(\ln x)}{x}$$

16. Evaluate $\int_1^4 \sqrt{x} - \frac{1}{x}\,dx$ exactly. Describe how it compares to your answer in #9. (10 points)

$$\int_1^4 x^{1/2} - \frac{1}{x}\,dx =$$

$$\frac{2}{3}x^{3/2} - \ln x \Big|_1^4 = \frac{2}{3}4^{3/2} - \ln 4 - \frac{2}{3}1^{3/2} + \cancel{\ln(1)}^{\,0} =$$

$$\frac{16}{3} - \ln 4 - \frac{2}{3} = \frac{14}{3} - \ln 4 \approx 3.28037$$

this value is a little less. #9 is a bit of an overestimate