1/18/2022

Chapter 2: Limits

2.1 Preview of Calculus

Differential calculus is concerned with rates of change. Velocity is a rate of change of position. Acceleration is a rate of change of velocity. Lots of functions and processes have rates of change that we might want to understand.

If the process is described by a straight line then the rate of change is constant. It's always changing at the same rate.

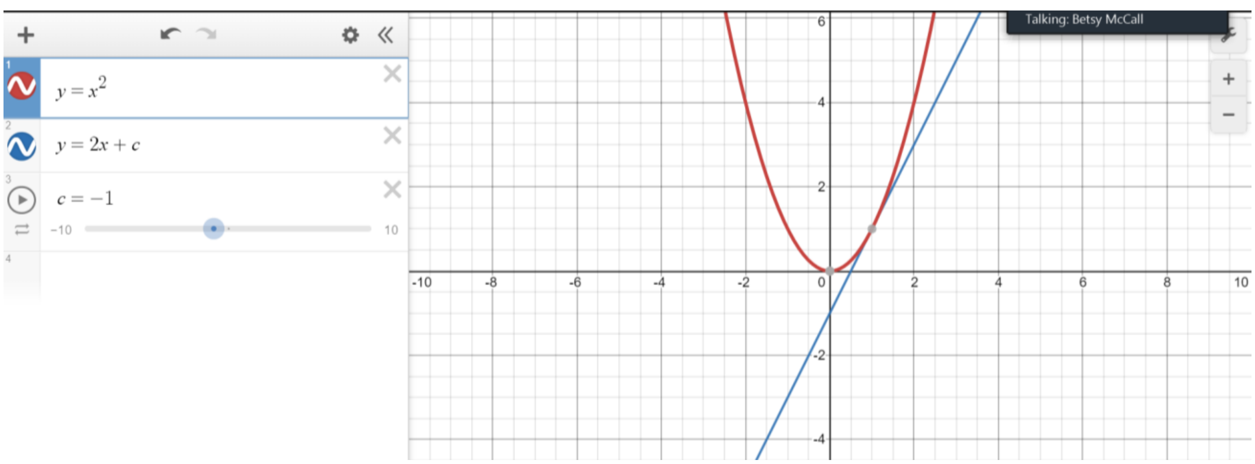
The rate of change of a line doesn't depend on where on the line you start, only how far in x you've moved.

When the line or function is curved, then the rate of change is changing. It does not depend on where you start and where you stop.

https://www.desmos.com/calculator

Secant line: is a line that connects two points on a graph

Tangent line: is a line that touches a graph only at one point, but as the same rate of change of the graph at that point.



Difference quotient

$$m_{sec} = \frac{f(x+h) - f(x)}{h}$$

$$m_{sec} = \frac{f(b) - f(a)}{b - a} = \frac{f(x) - f(a)}{x - a}$$

$b = x + h$, h is the distance between b and a.

Denominator $b - a = (x + h) - x = h$
We saw that in this formulation, we should be able to algebraically cancel the h.

These expressions are all equivalent to each other.

The average rate of change = difference quotient.

Suppose you want to find the average rate of change for the function $y = x^2$, between the points (-1,1) and (2,4). Average rate of change is $\frac{1-4}{-1-2} = -\frac{3}{-3} = 1$. (=slope of the secant line)

The slope of the tangent = instantaneous rate of change.
The average rate of change approximates the instantaneous rate of change as the points get closer together.

First part of calculus is about rates of change, use derivatives to find that, based on the difference quotient as the two points get closer together.
In the second part of calculus, we are going in the reverse direction: we will have the rates of change, and we will want to find total distance traveled, or the area under a curve.

Where are we going with this: start with an approximation and then find better approximations… and see where this is going as the approximations get better and better… in the limit.

2.2 Limits

Informally, as you get closer to a specific values in x, the limit is the value that the y-value approaches.

In our example, as x gets closer to 2, the y-value gets closer to 4 (even though the point (2,4) is not defined because in our function we divided by zero).

We will pick up on limits next time.