2/3/2022

3.3 Derivative Rules
3.4 Derivatives as Rates of Change
3.5 Derivatives of Trigonometric Functions

Constant Rule

$$\frac{d}{dx}[c] = 0$$

For $f(x) = c, f'(x) = 0$

Linear Function

$$\frac{d}{dx}[kx] = k$$

For $f(x) = kx, f'(x) = k$

$$\frac{d}{dx}[mx + b] = \frac{d}{dx}[mx] + \frac{d}{dx}[b] = m + 0 = m$$

Addition and Subtraction Rule

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$

Constant Multiplier Rule

$$\frac{d}{dx}[cf(x)] = c\left\{\frac{d}{dx}[f(x)]\right\} = cf'(x)$$

$$\frac{d}{dx}[kx] = k\left\{\frac{d}{dx}[x]\right\} = k(1) = k$$

Power Rule

$$\frac{d}{dx}[x^n] = nx^{n-1}, n \neq 0$$

$$f(x) = x^2$$

$$f'(x) = \lim_{h\to 0}\frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0}\frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h\to 0}\frac{2xh + h^2}{h}$$

$$= \lim_{h\to 0}\frac{h(2x+h)}{h} = \lim_{h\to 0} 2x + h = 2x$$

$$f(x) = x^3$$

$$f'(x) = \lim_{h\to 0}\frac{(x+h)^3 - x^3}{h} = \lim_{h\to 0}\frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} = \lim_{h\to 0}\frac{3x^2h + 3xh^2 + h^3}{h}$$

$$= \lim_{h\to 0} \frac{h(3x^2 + 3xh + h^2)}{h} = \lim_{h\to 0} 3x^2 + 3xh + h^2 = 3x^2$$

$$f(x) = x^4$$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^4 - x^4}{h} = \lim_{h\to 0} \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h}$$

$$= \lim_{h\to 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$$

$$= \lim_{h\to 0} \frac{h(4x^3 + 6x^2h + 4xh^2 + h^3)}{h} = \lim_{h\to 0} 4x^3 + 6x^2h + 4xh^2 + h^3 = 4x^3$$

$$f(x) = \frac{1}{x} = x^{-1}$$

$$\lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h\to 0} \frac{1}{h}\left[\frac{1}{x+h} - \frac{1}{x}\right] = \lim_{h\to 0} \frac{1}{h}\left[\frac{x}{(x+h)x} - \frac{x+h}{x(x+h)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{x-(x+h)}{(x+h)x}\right] = \lim_{h\to 0} \frac{1}{h}\left[\frac{x-x-h}{(x+h)x}\right] = \lim_{h\to 0} \frac{1}{h}\left[\frac{-h}{(x+h)x}\right] = \lim_{h\to 0} \left[\frac{-1}{(x+h)x}\right] = -\frac{1}{x^2} = -x^{-2}$$

$$f(x) = \sqrt{x} = x^{1/2}$$

$$\lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} = \lim_{h\to 0} \frac{\left(\sqrt{x+h}\right)^2 - \left(\sqrt{x}\right)^2}{h\left(\sqrt{x+h} + \sqrt{x}\right)} =$$

$$\lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h\to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h\to 0} \frac{1}{(\sqrt{x+h} + \sqrt{x})} = \frac{1}{(\sqrt{x} + \sqrt{x})} = \frac{1}{2\sqrt{x}}$$

$$f'(x) = \frac{1}{2}x^{1/2-1} = \frac{1}{2}x^{-1/2} = \frac{1}{2}\left[\frac{1}{x^{1/2}}\right] = \frac{1}{2}\left[\frac{1}{\sqrt{x}}\right] = \frac{1}{2\sqrt{x}}$$

Combining Rules

$$f(x) = 4x^3 + 3x - 11 + \frac{1}{\sqrt{x}}$$

$$f'(x) = 4\frac{d}{dx}[x^3] + 3\frac{d}{dx}[x] - \frac{d}{dx}[11] + \frac{d}{dx}\left[x^{-1/2}\right] =$$

$$4[3x^2] + 3[1] - 0 + \left(-\frac{1}{2}\right)x^{-\frac{3}{2}} = 12x^2 + 3 - \frac{1}{2\sqrt{x^3}}$$

Product Rule

$$\frac{d}{dx}[f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$h(x) = (x^2 + x)(x^3 - 5) = x^5 + x^4 - 5x^2 - 5x$$
$$h'(x) = 5x^4 + 4x^3 - 10x - 5$$

$$f(x) = x^2 + x, g(x) = x^3 - 5$$
$$f'(x) = 2x + 1, g'(x) = 3x^2$$

Using the product rule

$$h'(x) = (2x + 1)(x^3 - 5) + (x^2 + x)(3x^2) =$$
$$2x^4 - 10x + x^3 - 5 + 3x^4 + 3x^3 = 5x^4 + 4x^3 - 10x - 5$$

But what about $f'(x) \cdot g'(x)$?

$$(2x + 1)(3x^2) = 6x^3 + 3x^2$$

It doesn't even have the same number of terms, and the power isn't right. It doesn't work.

Find the derivative of $h(x) = \left(x^4 - \frac{3}{x^2} + \sqrt[3]{x^5}\right)\left(x - x^{11} + 81 - \frac{6}{x^\pi}\right)$

$f(x) = x^4 - \frac{3}{x^2} + \sqrt[3]{x^5} =$ $x^4 - 3x^{-2} + x^{5/3}$	$g(x) = x - x^{11} + 81 - \frac{6}{x^\pi}$ $= x - x^{11} + 81 - 6x^{-\pi}$
$f'(x) = 4x^3 + 6x^{-3} + \frac{5}{3}x^{2/3}$ $= 4x^3 + \frac{6}{x^3} + \frac{5\sqrt[3]{x^2}}{3}$	$g'(x) = 1 - 11x^{10} + 6\pi x^{-\pi-1}$ $= 1 - 11x^{10} + \frac{6\pi}{x^{\pi+1}}$

$$h(x) = \left(x^4 - \frac{3}{x^2} + \sqrt[3]{x^5}\right)\left(1 - 11x^{10} + \frac{6\pi}{x^{\pi+1}}\right) + \left(4x^3 + \frac{6}{x^3} + \frac{5\sqrt[3]{x^2}}{3}\right)\left(x - x^{11} + 81 - \frac{6}{x^\pi}\right)$$

In the MyOpenMath homework, you may have to simplify at this point. On exams and quizzes, I will not expect you to go past this point.

$$f(x) = (6x^2 - 11x)^2 = (6x^2 - 11x)(6x^2 - 11x)$$

Can use the product rule to get around powers (until we get to the chain rule).

Product rule can be extended to any number of terms.

$$s(x) = f(x)g(x)h(x)$$

$$s'(x) = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$$

$$s(x) = f(x)[g(x)h(x)]$$

$$s'(x) = f'(x)[g(x)h(x)] + f(x)[g(x)h(x)]' =$$
$$f'(x)[g(x)h(x)] + f(x)[g'(x)h(x) + g(x)h'(x)]$$

Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$$

$$h(x) = \frac{x^2 + 1}{x^3 - 5x - 2}$$

$f(x) = x^2 + 1$	$g(x) = x^3 - 5x - 2$
$f'(x) = 2x$	$g'(x) = 3x^2 - 5$

$$h'(x) = \frac{2x(x^3 - 5x - 2) - (x^2 + 1)(3x^2 - 5)}{(x^3 - 5x - 2)^2}$$

In general, you don't need to simplify, but it is somewhat expected that your answers will be in a similar form to the original problem.

Next week we'll look at word problems (3.4 rates of change), and trig function derivatives.