3/24/2022

4.9 Newton's Method
4.10 Antiderivatives

Newton's method is an iterative method using the derivative to approximate the value of zero (real zeros).

The function is continuous and differentiable (at least near the zero you are estimating)
Sometimes the method will not converge. If that happens, pick a different initial guess and try again.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Find the zero(s) of the polynomial $f(x) = x^3 - 3x + 1$.
Begin with a guess about the approximate value of the zero. $x_0 = 2$?
Find the derivative.

$$f'(x) = 3x^2 - 3$$

(this function has critical points at ± 1, so don't pick those values as the starting point since you can't divide by zero).

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

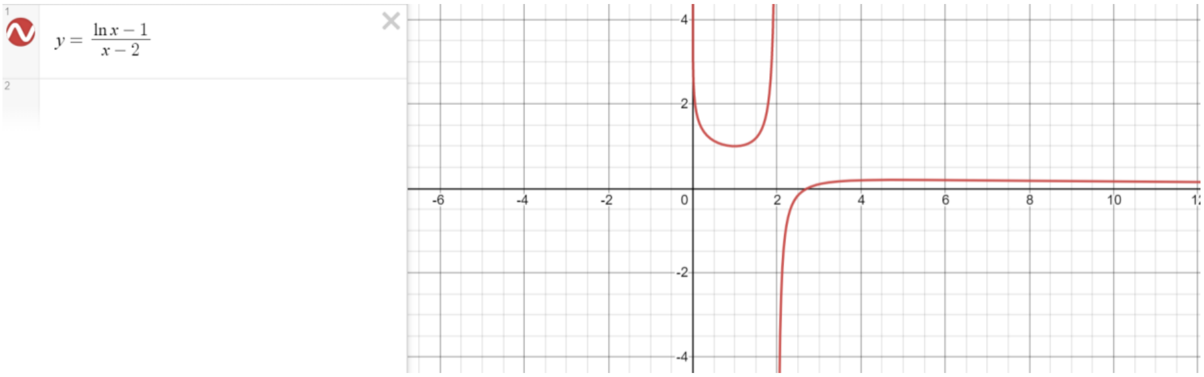
$$x_1 = 2 - \frac{[2^3 - 3(2) + 1]}{[3(2^2) - 3]}$$

Whatever this value is, it becomes the new x_n and we repeat the formula to find x_2. Keep going until the value stabilizes (stop when the 4th decimal place stops changing). (Within an error of ± 0.001).

n	x_n	f(x_n)	f'(x_n)	x_n+1
0	2	3	9	1.666667
1	1.666667	0.62963	5.333333	1.548611
2	1.548611	0.06804	4.194589	1.53239
3	1.53239	0.001218	4.044659	1.532089
4	1.532089	4.17E-07	4.04189	1.532089
5	1.532089	4.88E-14	4.041889	1.532089
6	1.532089	0	4.041889	1.532089
7	1.532089	0	4.041889	1.532089

We found our zero by x_4

Find the zero of the function $f(x) = \frac{\ln x - 1}{x - 2}$.



$$f'(x) = \frac{\left(\frac{1}{x}\right)(x-2) - (1)(\ln x - 1)}{(x-2)^2}$$

Start with initial guesses: 1.5, 2.1, 5

n	x_n	f(x_n)	f'(x_n)	x_n+1
0	2.1	-2.58063	30.56817	2.184422
1	2.184422	-1.18559	8.910954	2.317471
2	2.317471	-0.50248	2.941969	2.488269
3	2.488269	-0.18107	1.193932	2.639931
4	2.639931	-0.0457	0.663356	2.708829
5	2.708829	-0.00491	0.527741	2.718141
6	2.718141	-7.2E-05	0.512393	2.718282
7	2.718282	-1.6E-08	0.512166	2.718282

The section of the graph with 1.5, has no zero and so we ended up not converging and then getting a value that was not in the domain.

When we tried 5, the method didn't converge (it blew up) and so didn't find the zero.

The guess at 2.1 did.

Zeros at critical points (places where the graph touches the axis but doesn't pass through it).

Antiderivatives (4.10)

Since chapter 3 we've been finding derivatives. And now, we are going to be given the derivative and asked to find the original function.

What function produces the given function as its derivative?

The antiderivative is often given the capital version of the function name.
So if $f(x)$ is the function, then it's antiderivative is $F(x)$. ie. $F'(x) = f(x)$

Integral symbol is used to indicate that we need to find the (an) antiderivative of a given function.

$$\int f(x)\,dx = F(x)$$

Indefinite integral. (definite integral is in Chapter 5)

Functions do not have only one antiderivative.

$$F(x) = 3x^2 + 11$$
$$G(x) = 3x^2 - 5$$

$$F'(x) = f(x) = 6x$$
$$G'(x) = g(x) = 6x$$

When we take a derivative, we lose information about the constants. And that means we can't recover them when we go backwards.

$$f(x) = g(x)$$

But $F(x) \neq G(x)$. So, antiderivatives can differ by a constant.

We would need additional information to figure out what that constant is (such as a point on the curve).

$$\int 6x\,dx = 3x^2 + C$$

What this tells us is that antiderivative is actually a family of functions that differ only by that constant. Any term in the original function that contains an x can be recovered, but the constant that has no x is zero and so it's the same for every constant, so it can't be recovered.

Power rule for antiderivatives:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$$

$$F(x) = x^3$$
$$F'(x) = 3x^2$$

$$f(x) = 3x^2$$

$$3\int x^2 dx = 3\left(\frac{x^3}{3}\right) + C = x^3 + C$$

On page 497, there is a list of antiderivative rules.

$$\int k\,dx = \int kx^0 dx = kx^1 + C = kx + C$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int e^x dx = e^x + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \frac{1}{1+x^2} dx = \arctan x + C = \tan^{-1} x + C$$

It is more complicated to do antiderivatives in general than to do derivatives.

Some things do the same. For example, if you have a function multiplied by a constant, you can pull it outside the integral and then use the rules on just the x part of the function.

You can do antiderivatives term by term.

Find $\int 8\, sec\, x(\sec x - 4\tan x)dx$

$$\int 8\sec^2 x - 32\sec x \tan x \, dx = \int 8\sec^2 x \, dx - \int 32 \sec x \tan x \, dx$$

$$= 8\int \sec^2 x \, dx - 32\int \sec x \tan x \, dx$$

$$= 8\tan x - 32\sec x + C$$

Initial Value problems are how the constant information.

I'm given $f'(x) = x^3 - 8x^2 + 16x + 1, f(0) = 0$
Another version: $F(x) = \int x^3 - 8x^2 + 16x + 1dx\,, F(0) = 0$

$$\int x^3 - 8x^2 + 16x + 1dx = \frac{x^4}{4} - 8\left(\frac{x^3}{3}\right) + 16\left(\frac{x^2}{2}\right) + 1(x) + C$$

$$f(x) = \frac{1}{4}x^4 - \frac{8}{3}x^3 + 8x^2 + x + C$$

$$f(0) = \frac{1}{4}(0)^4 - \frac{8}{3}(0)^3 + 8(0)^2 + 0 + C = 0$$
$$C = 0$$

$$f(x) = \frac{1}{4}x^4 - \frac{8}{3}x^3 + 8x^2 + x$$

Be especially careful with functions that are not 0 at x=0 ($e^x, \cos x, etc.$)

You can take more than one antiderivative.

$a(t)$ this is the second derivative of the position function, but it's the first derivative of velocity.

$$\int a(t)\,dt = v(t) + C$$

$v(t)$ is the derivative of position, so the antiderivative of velocity is position.

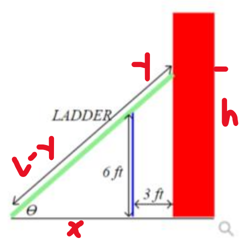
$$\int v(t)dt = s(t) + K$$

We need initial conditions to solve for the constant. We need one condition for each constant.

Another optimization example.

Question 5 0/10 pts 100 99 Details

A fence 6 feet tall runs parallel to a tall building at a distance of 3 ft from the building as shown in the diagram.



We wish to find the length of the shortest ladder that will reach from the ground over the fence to the wall of the building.

[A] First, find a formula for the length of the ladder in terms of θ. (*Hint: split the ladder into 2 parts.*) Type **theta** for θ.

$L(\theta) =$

$$L(\theta) =$$

$$\frac{base}{hypotenuse} = \frac{x+3}{L} = \frac{x}{L-y}$$

$$\frac{vertical\ height}{base} = \frac{h}{x+3} = \frac{6}{x}$$

Properties of similar triangles more-or-less work the same as trig functions.

$$\sin(\theta) = \frac{6}{L-y}$$
$$\sin(\theta) = \frac{h}{L}$$

$$L\sin(\theta) = h$$

These are equal to each other.

$$\cos(\theta) = \frac{x}{L-y}$$
$$\cos(\theta) = \frac{x+3}{L}$$

$$L\cos(\theta) = x+3$$
$$x = L\cos(\theta) - 3$$

$$\frac{h}{x+3} = \frac{6}{x}$$

$$\frac{L\sin(\theta)}{L\cos(\theta)} = \frac{6}{L\cos(\theta) - 3}$$

$$\tan(\theta) = \frac{6}{L\cos(\theta) - 3}$$

$$\frac{(L\cos\theta - 3)\sin(\theta)}{\cos(\theta)} = 6$$

$$L - \frac{3\sin\theta}{\cos\theta} = 6$$

$$L - 3\tan\theta = 6$$

$$L = 6 + 3\tan(\theta)$$

Assuming I didn’t make any arithmetic errors.